Strong phase in $D \to K\pi$ decays and CP asymmetry in four-body D decays

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Outline

- Strong phase in $D \to K\pi$ decays
- CP violation in four-body D decays
- Summary

Strong phase

- Effective Hamiltonian for $D \to K\pi$ decays $(c \to su\bar{d},\ \ c \to du\bar{s})$

$$
\begin{aligned}
\mathcal{H}_{\rm eff} \;=\; & \frac{G_F}{\sqrt{2}}\{V_{ud}V_{cs}^*[C_1(\bar{s}_i c_i)_{V-A}(\bar{u}_j d_j)_{V-A} + C_2(\bar{s}_i c_j)_{V-A}(\bar{u}_j d_i)_{V-A}] \\
& + V_{us}V_{cd}^*[C_1(\bar{d}_i c_i)_{V-A}(\bar{u}_j s_j)_{V-A} + C_2(\bar{d}_i c_j)_{V-A}(\bar{u}_j s_i)_{V-A}]\} \\
& + \mathrm{H.c.},
\end{aligned}
$$

where the first line governs *Cabibbo favored* (CF) decays:

$$D^0 \to K^-\pi^+, \quad D^0 \to \bar{K}^0\pi^0, \quad D^+ \to \bar{K}^0\pi^+,$$

and the second line *doubly Cabibbo suppressed* (DCS) decays:

$$D^0 \to K^+\pi^-, \quad D^0 \to K^0\pi^0, \quad D^+ \to K^0\pi^+, \quad D^+ \to K^+\pi^0.$$

NO penguin contributions in $D \to K\pi$ *decays* !

Some experimental observables

- Asymmetries for $D \to K_S\pi$ vs. $D \to K_L\pi$

An interesting asymmetry due to interference between CF $D \to \overline{K^0}\pi$ and DCS $D \to K^0\pi$ decays, defined as

$$R(D) \equiv \frac{\mathcal{B}(D \to K_S\pi) - \mathcal{B}(D \to K_L\pi)}{\mathcal{B}(D \to K_S\pi) + \mathcal{B}(D \to K_L\pi)},$$

(Bigi and Yamamoto, 1995)

have been observed by CLEO Collaboration

$$R(D^0) = 0.108 \pm 0.025 \pm 0.024,$$

$$R(D^+) = 0.022 \pm 0.016 \pm 0.018.$$

(Q. He *et al.*, CLEO Collaboration, 2008)

Theoretically, these asymmetries

$$R(D) = \frac{2r_D \cos\delta_D}{1 + r_D^2},$$

where we parameterize

$$\frac{A(D \to K^0\pi)}{A(D \to \overline{K^0}\pi)} = -r_D e^{-i\delta_D},$$

Here r_D is real and δ_D is the strong phase (neglecting CP violation effects)

- $\delta_{K\pi}$

The relative strong phase $\delta_{K\pi}$ between $D^0 \to K^-\pi^+$ and $D^0 \to K^+\pi^-$

$$\frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} = -R_D e^{i\delta_{K\pi}}$$

has been also reported by CLEO Collaboration

$$\cos\delta_{K\pi} = 1.03^{+0.31}_{-0.17} \pm 0.06$$

(Asner *et al.*, CLEO Collaboration, 2008)

with very large uncertainty.

The phase is important in the search for $D^0 - \bar{D}^0$ mixing, since it appears in the time dependence assuming CP invariance

$$R(t) = [R_D^2 + R_D y'\Gamma t + (y'^2 + x'^2)(\Gamma t)^2/4]e^{-\Gamma t}$$

with $x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi}$, $y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$ and $x = \Delta m/\Gamma$, $y = \Delta\Gamma/2\Gamma$ are two dimensionless parameters describing $D^0 - \bar{D}^0$ mixing.

$\Longrightarrow$

A difficult task:

To calculate the amplitude $A(D \to K\pi)$ including the strong phases

$\Longrightarrow$

(i) Naive factorization cannot work (also due to large color-suppressed amplitudes).

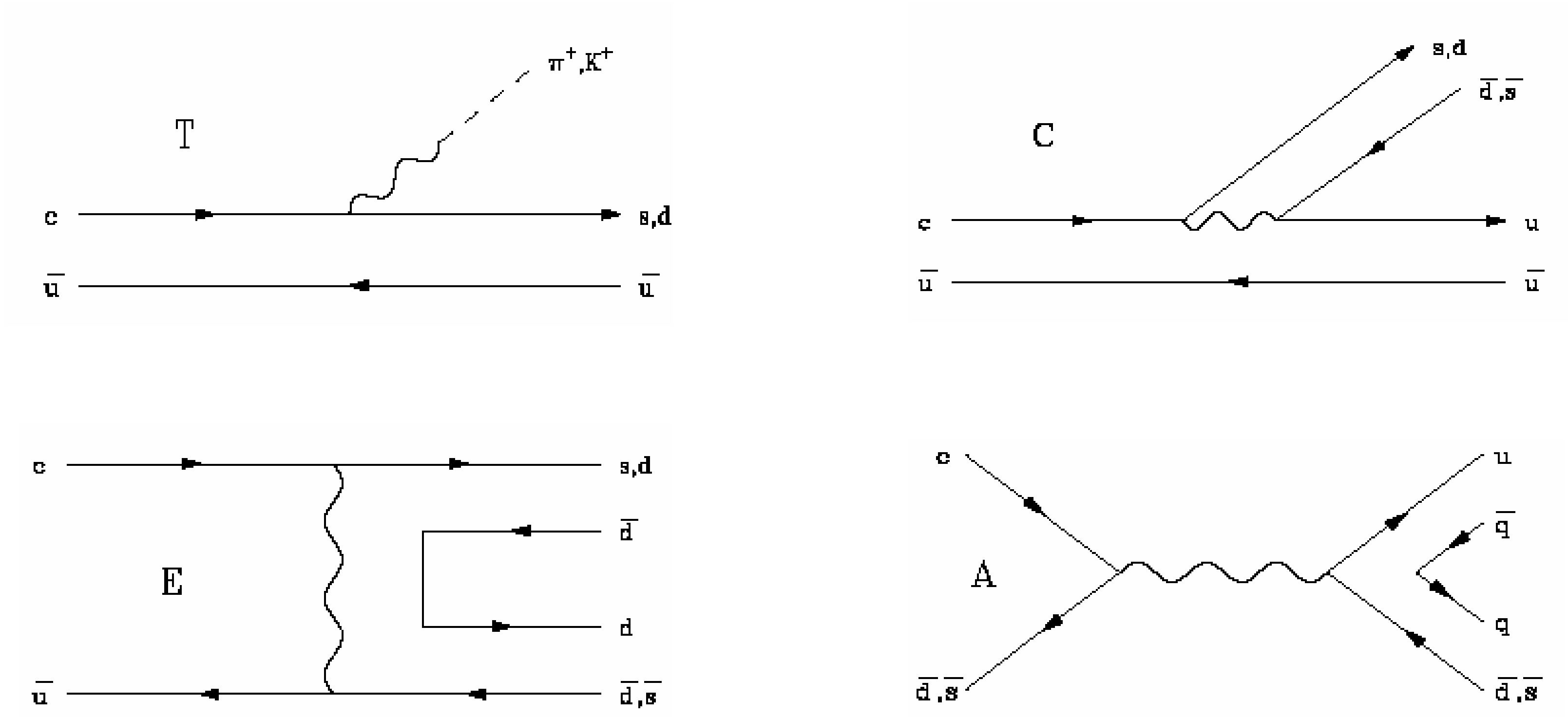
(ii) QCD factorization and pQCD approaches, which work in B decays, cannot be expected to work here since m_c is not heavy enough.

$\Downarrow$

Phenomenological Analysis for non-leptonic D decays

- Amplitude decomposition and theoretical assumptions

In terms of the quark-diagram topologies: $\mathcal{T}$ (color-allowed), $\mathcal{C}$ (color-suppressed), $\mathcal{E}$ (W-exchange), and $\mathcal{A}$ (W-annihilation)



(Chau and Cheng, PRL 56(1986)1655; PRD 36 (1987) 137)

$$A(D^0 \to K^-\pi^+) = i\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*(\mathcal{T}+\mathcal{E}), \quad \sqrt{2}A(D^0 \to \bar{K}^0\pi^0) = i\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*(\mathcal{C}-\mathcal{E}),$$

$$A(D^+ \to \bar{K}^0\pi^+) = i\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*(\mathcal{T}+\mathcal{C}),$$

$$A(D^0 \to K^+\pi^-) = i\frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*(\mathcal{T}'+\mathcal{E}'), \quad \sqrt{2}A(D^0 \to K^0\pi^0) = i\frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*(\mathcal{C}'-\mathcal{E}'),$$

$$A(D^+ \to K^0\pi^+) = i\frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*(\mathcal{C}'+\mathcal{A}'), \quad \sqrt{2}A(D^+ \to K^+\pi^0) = i\frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*(\mathcal{T}'-\mathcal{A}').$$

In general $\mathcal{T}$, $\mathcal{C}$, $\mathcal{E}$, and $\mathcal{A}$ could have non-trivial strong phases.

$\Longrightarrow$ only using the available experimental data, it is impossible to determine these amplitudes without any theoretical assumptions.

Factorization hypothesis $\Longrightarrow$

$$\frac{\mathcal{T}'}{\mathcal{T}} = \frac{f_K}{f_\pi}\frac{(m_D^2 - m_\pi^2)F_0^{D\to\pi}(m_K^2)}{(m_D^2 - m_K^2)F_0^{D\to K}(m_\pi^2)}, \quad \mathcal{C} = \mathcal{C}'$$

$$\mathcal{E} = \mathcal{E}', \;\; \mathcal{A}' = \frac{C_2}{C_1}\mathcal{E}'$$

(Gao, 2007)

Now we reduce independent complex phenomenological parameters appearing in decay amplitudes as $\mathcal{T}$, $\mathcal{C}$, and $\mathcal{E}$. Five branching ratios of $D \to K\pi$ decays are measured up to now, this enables us to determine the $D \to K\pi$ amplitudes from the present data.

Although these are not model independent relations, one will find that phenomenologically they work very well in $D \to K\pi$ decays.

Phenomenological results

(Gao, 2007)

- The asymmetry $R(D^0)$

$\mathcal{C} = \mathcal{C}', \mathcal{E} = \mathcal{E}' \Longrightarrow$

$$A(D^0 \to K^0\pi^0) = \frac{V_{us}V_{cd}^*}{V_{ud}V_{cs}^*} A(D^0 \to \bar{K}^0\pi^0) = -\tan^2\theta_C A(D^0 \to \bar{K}^0\pi^0).$$

This means that $\delta_{D^0} = 0^\circ$. Consequently,

$$R(D^0) = \frac{2\tan^2\theta_C}{1+\tan^4\theta_C} \simeq 2\tan^2\theta_C.$$

consistent with the result using U-spin relation

(Bigi, Yamamoto, 1995; Rosner, 2006).

$\tan\theta_C \simeq 0.23 \Longrightarrow$

$$R(D^0) \simeq 0.106$$

in agreement with the measurement $R(D^0) = 0.108 \pm 0.025 \pm 0.024$

- The asymmetry $R(D^+)$

$$\frac{A(D^+ \to K^0\pi^+)}{A(D^+ \to \bar{K}^0\pi^+)} = -\tan^2\theta_C \frac{\mathcal{C}' + \mathcal{A}'}{\mathcal{C} + \mathcal{T}} = -\tan^2\theta_C \frac{\mathcal{C} + C_2/C_1\mathcal{E}}{\mathcal{C} + \mathcal{T}},$$

Numerically

$$\frac{A(D^+ \to K^0\pi^+)}{A(D^+ \to \bar{K}^0\pi^+)} = \begin{cases} -\tan^2\theta_C\ 1.538e^{\pm i106^\circ}, & C_2/C_1 = -0.3, \\ -\tan^2\theta_C\ 1.532e^{\pm i105^\circ}, & C_2/C_1 = -0.4, \\ -\tan^2\theta_C\ 1.521e^{\pm i103^\circ}, & C_2/C_1 = -0.5, \end{cases}$$

here no similar U-spin relation for the charged case here it is found that δ_{D+} is about 100°. This leads to

$$R(D^+) = \begin{cases} 0.044, & C_2/C_1 = -0.3, \\ 0.040, & C_2/C_1 = -0.4, \\ 0.035, & C_2/C_1 = -0.5. \end{cases}$$

$R(D^+) = -0.005 \pm 0.013$ was obtained theoretically also by (Bhattacharya and Rosner, 2010)

The present observed value by CLEO Collaboration

$$R(D^+) = 0.022 \pm 0.016 \pm 0.018.$$

- Estimation of $\delta_{K\pi}$

C_2/C_1	-0.2	-0.3	-0.4	-0.5	-0.6
$\cos\delta_{K\pi}$	0.983±0.015	0.980±0.017	0.976±0.021	0.970±0.026	0.960±0.034
$\delta_{K\pi}$	$10.5^\circ \pm 4.6^\circ$	$11.4^\circ \pm 4.9^\circ$	$12.6^\circ \pm 5.5^\circ$	$14.1^\circ \pm 6.1^\circ$	$16.2^\circ \pm 6.9^\circ$

A not large but nonzero $\delta_{K\pi}$, whose magnitude is 10° or above, i.e. $\sin\delta \sim \pm 0.2$, might be expected from the present analysis.

- Some other theoretical estimations

By assuming the existence of nearby resonances for the D meosn, $\sin\delta_{K\pi} = \pm 0.31$, i.e. $\delta_{K\pi} = \pm 18^\circ$ was obtained. (A.F. Falk,Y. Nir, A. Petrov, 1999)

The other existing hadronic models which incorporate SU(3) symmetry breaking effects seems to prefer a small value of this phase with most models giving $\sin\delta \leq 0.1$.

(T.E. Browder, S. Pakvasa, 1996)

Four-body D decays

$D^0 \to K^-\pi^+\ell^+\ell^-$, $D^0 \to \pi^+\pi^-\ell^+\ell^-$,

$\ell = e,\ \mu$

$D^0 \to K^+K^-\ell^+\ell^-$, $D^0 \to K^+\pi^-\ell^+\ell^-$.

Cappiello, Cata, D'Ambrosio,2012

$D \to \bar{K}K\pi^+\pi^-$

H.B. Li, 0902.3032[hep-ex]
X.W. Kang and H.B. Li, 2010

$\Longrightarrow$ CP violation effects

Proposed by Bigi

Motivated by the CP study in $K_L \to \pi^+\pi^-e^+e^-$

$$K_L \to \pi^+ \pi^- e^+ e^-$$

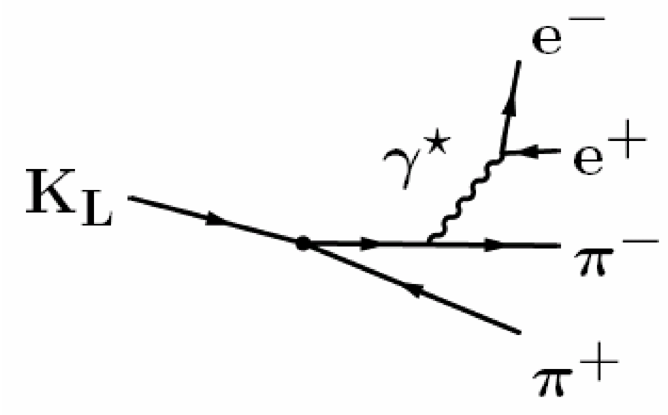
Sehgal et al published in 1992 — 2000

Amplitude has two major contributions:

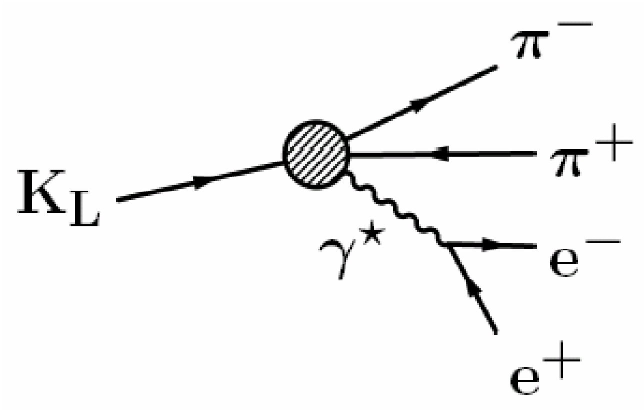
Inner Bremssstrahlung (IB) , electric

Direct Emission (DE) , magnetic + small electric

IB CPV



DE
- M1 CP conserving
- E1 CPV very small



Thus the amplitude of $K_L(p) \to \pi^+(p_+)\pi^-(p_-)e^+(k_+)e^-(k_-)$

can be written as

$$\mathcal{M}(K_L \to \pi^+\pi^-e^+e^-) = \frac{e}{q^2}\bar{u}(k_-)\gamma_\mu v(k_+)V^\mu$$

$$V_\mu = iM\varepsilon^{\mu\nu\alpha\beta}p_{+\nu}p_{-\alpha}q_\beta + E_+p_+^\mu + E_-p_-^\mu$$

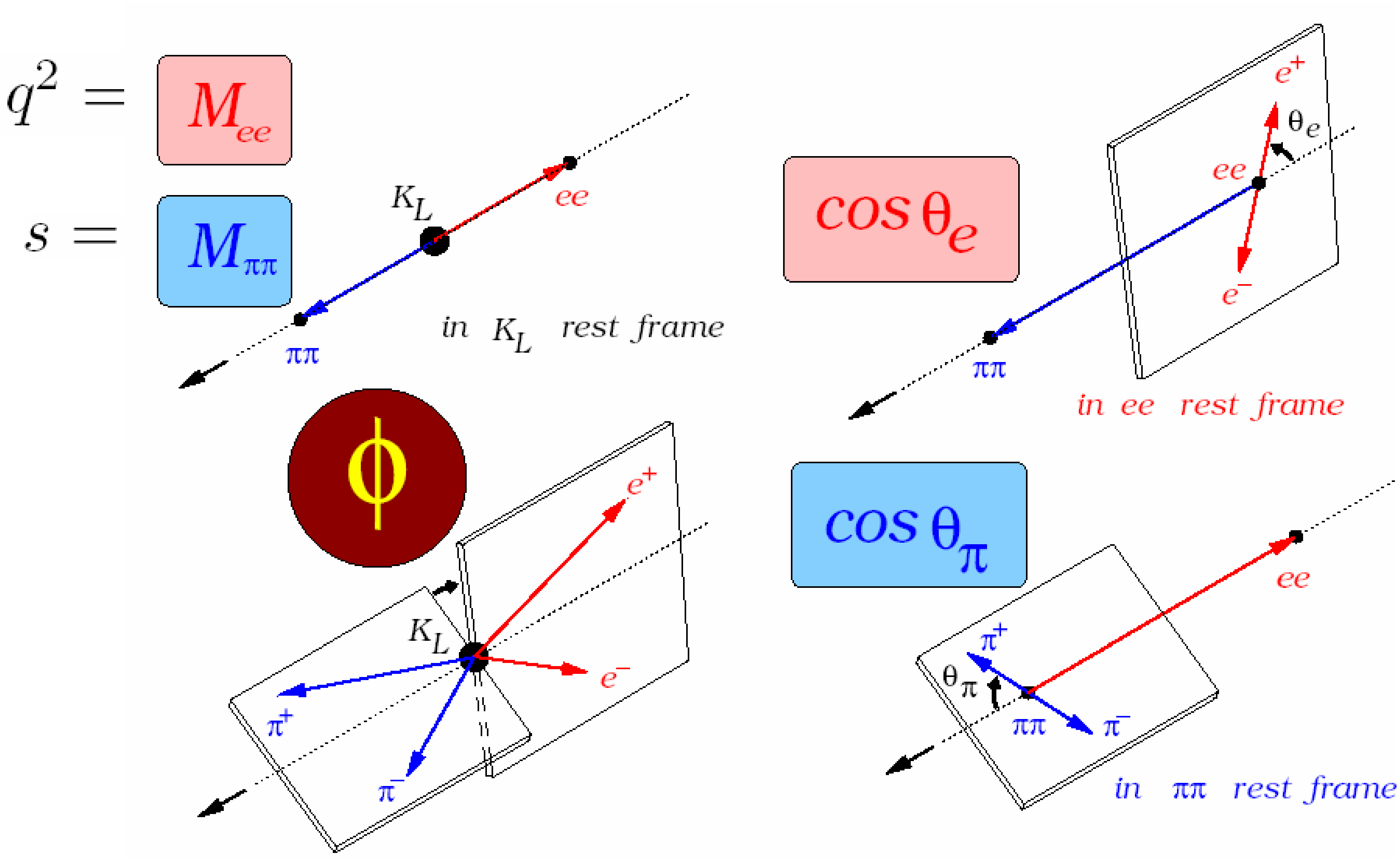
$$q^2 = (k_+ + k_-)^2 = M_{ee}$$

$$s = (p_+ + p_-)^2 = M_{\pi\pi}$$

The Lorentz invariant form factors M and $E_\pm$ stand for the magnetic and electric transitions, respectively, depending on scalar products of q , p_+ and p_-, which can be calculated in chiral perturbation theory.

Kinematics

Five independent kinematic varibles:



The range of the variables is:

$$4m_\pi^2 \leq s \leq (M_K - 2m_e)^2$$

$$4m_e^2 \leq q^2 \leq (M_K - \sqrt{s})^2$$

$$0 \leq \theta_\pi,\ \theta_e \leq \pi$$

$$0 \leq \phi \leq 2\pi$$

Angle ϕ will play important role in CP violation

Now the differential decay rate takes the form

$$d\Gamma = I(s, q^2, \cos\theta_e, \cos\theta_\pi, \phi)\, ds\, dq^2\, d\cos\theta_e\, d\cos\theta_\pi\, d\phi$$

where I is from squaring the amplitude.

The differential decay rate with respect to ϕ, after integration over other four variables, will be in a general form as

$$\frac{d\Gamma}{d\phi} = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \sin\phi\cos\phi$$

$$\Downarrow$$

We will see

The angular distribution will give a CP-violating asymmetry, which is due to interference of CP-even and CP-odd parts.

ϕ is defined, in the K_L rest frame, as

$$\sin\phi = \vec{n}_\pi \times \vec{n}_l \cdot \vec{z}\,, \quad \cos\phi = \vec{n}_\pi \cdot \vec{n}_l$$

with

$$\vec{n}_\pi = (\vec{p}_+ \times \vec{p}_-)/|\vec{p}_+ \times \vec{p}_-|,$$
$$\vec{n}_l = (\vec{k}_+ \times \vec{k}_-)/|\vec{k}_+ \times \vec{k}_-|,$$
$$\vec{z} = (\vec{p}_+ + \vec{p}_-)/|\vec{p}_+ + \vec{p}_-|.$$

under C

$$\vec{p}_\pm \longrightarrow \vec{p}_\mp$$
$$\vec{k}_\pm \longrightarrow \vec{k}_\mp$$

$\longrightarrow$

under CP

$$\sin\phi \longrightarrow -\sin\phi$$
$$\cos\phi \longrightarrow \cos\phi$$

$\longleftarrow$

under P

$$\vec{p}_\pm \longrightarrow -\vec{p}_\pm$$
$$\vec{k}_\pm \longrightarrow -\vec{k}_\pm$$

$\Longrightarrow$ $\sin\phi\cos\phi$ is a CP-odd quantity!!

CP-violating asymmetry is defined as

$$\mathcal{A}_{\rm CP} = \frac{N_{\sin\phi\cos\phi>0.0} - N_{\sin\phi\cos\phi<0.0}}{N_{\sin\phi\cos\phi>0.0} + N_{\sin\phi\cos\phi<0.0}}$$

$$= \frac{\int_0^{2\pi} \frac{d\Gamma}{d\phi}\, d\phi\ \mathrm{sign}(\sin\phi\cos\phi)}{\int_0^{2\pi} \frac{d\Gamma}{d\phi}\, d\phi} \propto \Gamma_3$$

$$\propto \mathrm{Im}[(E_+ - E_-)M^*]$$

$$\int_0^{2\pi} \frac{d\Gamma}{d\phi}\, d\phi\ \mathrm{sign}(\sin\phi\cos\phi) = \left(\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi}\right) \frac{d\Gamma}{d\phi}\, d\phi$$

A large CP violating asymmetry

The predicted value is

$$|\mathcal{A}_{\rm CP}| \simeq 14\%$$

Heiliger and Sehgal,PRD (1993)

To be compared with the observed value by KTeV and NA48

$$|\mathcal{A}_{\rm CP}| = (13.7 \pm 1.5)\%$$

(PDG 2006)

$$B(K_L \to \pi^+\pi^- e^+ e^-)$$

$$= (1.3 \times 10^{-7})_{\rm IB} + (1.8 \times 10^{-7})_{\rm M1} + (0.04 \times 10^{-7})_{\rm CR}$$

$$\simeq 3.1 \times 10^{-7}$$ Theory by Heiliger and Sehgal (1993)

$$= (3.11 \pm 0.19) \times 10^{-7}$$ Data From PDG06 (NA48 and KTeV)

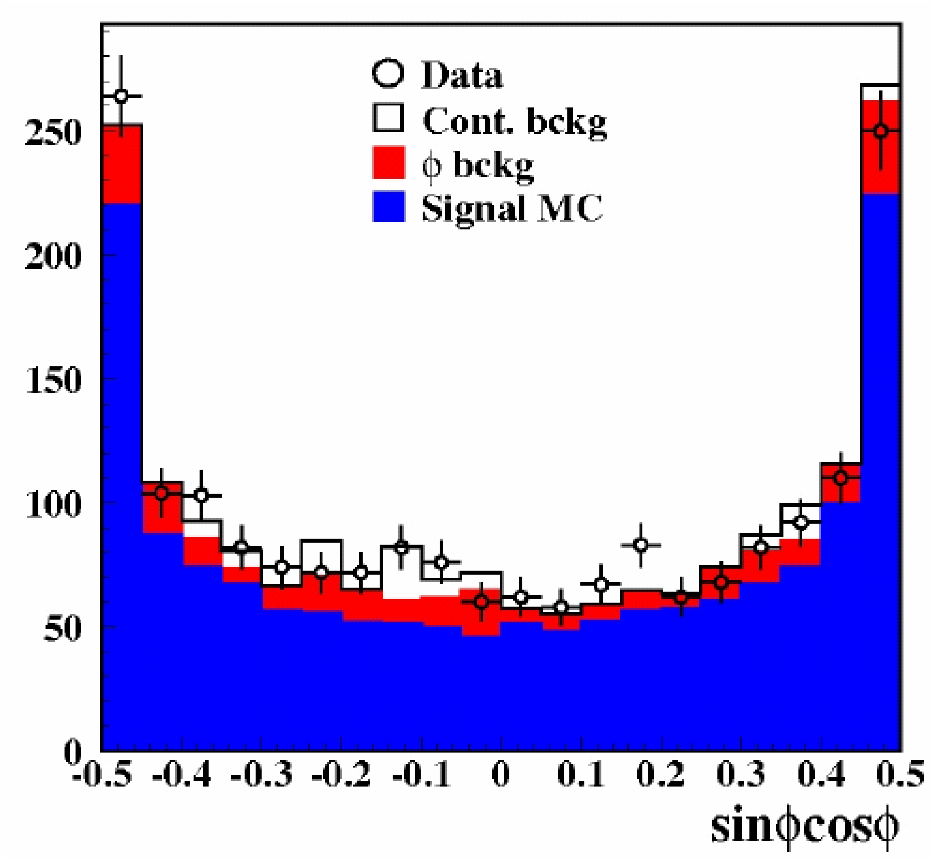
The comparable IB and M1 contributions lead to a large CP-violating asymmetry

IB suppressed by CP symmetry
M1 CPC but $O(p^4)$ in CHPT

Similar analysis of CP violating asymmetry in $\eta \to \pi^+\pi^- e^+ e^-$ has been done in (Gao, 2002)

$$\mathcal{A}_{\rm CP} = (-0.6 \pm 2.5_{\rm stat} \pm 1.8_{\rm syst}) \cdot 10^{-2}$$

by KLOE Collaboration 2009



Therefore, the search for CP violation (confirm or rule out) in this decay clearly requires much more data

Similar study in $\eta' \to \pi^+\pi^- e^+ e^-$ through $J/\psi \to \gamma\eta'$

Summary

- Strong phase in $D \to K\pi$ decays plays important roles in some interesting observables, which are related to asymmetries and the D-mixing parameters. precise measurements of them will be welcome both theoretically and experimentally.
- Four-body D decays or $\eta' \to \pi^+\pi^-\ell^+\ell^-$ may induce the CP violating effects. Large CPV in D sector may signal new physics.
- These could be interesting topics in STCF.

Thank you !

Angular distribution of $\tau^- \to K_S \pi^- \nu_\tau$ decay

Dao-Neng Gao and Xian-Fu Wang, Phys.Rev.D 87,073016(2013)

Study of $\tau^- \to K_S \pi^- \nu_\tau$ decay at Belle

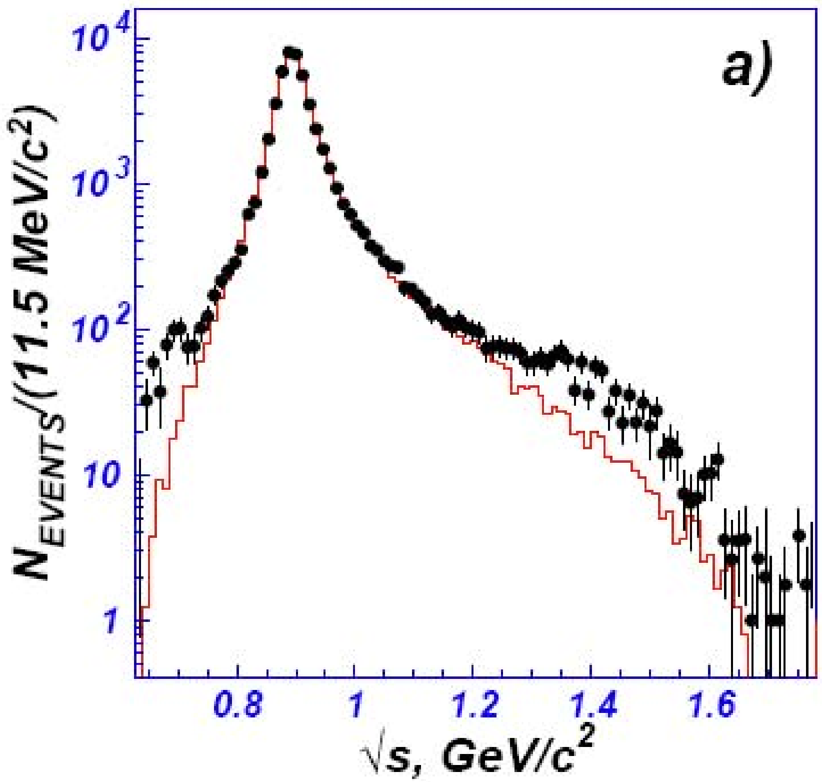


Figure 1: Comparison of the $K_S\pi$ mass distributions, points are experimental data, histogram is the fitted result with the model incorporating the $K^*(892)$ alone.

The vector form factor F_V is parameterized by the $K^*(892)$, $K^*(1410)$ and $K^*(1680)$ meson amplitudes:

$$F_V = \frac{1}{1+\beta+\chi}\left[BW_{K^*(892)}(s)+\beta BW_{K^*(1410)}(s)+\chi BW_{K^*(1680)}(s)\right], \tag{20}$$

where β and χ are complex coefficients for the fractions of the $K^*(1410)$ and $K^*(1680)$ resonances, respectively. $BW_R(s)$ is the relativistic Breit-Wigner function:

$$BW_R(s) = \frac{M_R^2}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)}, \tag{21}$$

where $\Gamma_R(s)$ is the s-dependent total width of the resonance:

$$\Gamma_R(s) = \Gamma_{0R}\frac{M_R^2}{s}\left(\frac{P(s)}{P(M_R^2)}\right)^{2\ell+1}, \tag{22}$$

where $\ell = 1(0)$ if the $K\pi$ system originates in the $P(S)$-wave state and Γ_{0R} is the resonance width at its peak.

The scalar form factor F_S includes the $K_0^*(800)$ and $K_0^*(1430)$ contributions, their fractions are described respectively by the complex constants $\varkappa$ and γ:

$$F_S = \varkappa \frac{s}{M^2_{K_0^*(800)}} BW_{K_0^*(800)}(s) + \gamma \frac{s}{M^2_{K_0^*(1430)}} BW_{K_0^*(1430)}(s). \quad (23)$$

	$K^*(892)$	$K_0^*(800) + K^*(892)$ $+K^*(1410)$	$K_0^*(800) + K^*(892)+$ $+K^*(1680)$
$M_{K^*(892)^-}$	895.53 ± 0.19	895.47 ± 0.20	894.88 ± 0.20
$\Gamma_{K^*(892)^-}$	49.29 ± 0.46	46.19 ± 0.57	45.52 ± 0.51
$\|\beta\|$		0.075 ± 0.006	
$\arg(\beta)$		1.44 ± 0.15	
$\|\chi\|$			$0.117 \pm$ 0.017 0.033
$\arg(\chi)$			3.17 ± 0.47
$\varkappa$		1.57 ± 0.23	1.53 ± 0.24
$\chi^2/\text{n.d.f.}$	$448.4/87$	$90.2/84$	$106.8/84$
$P(\chi^2), \%$	0	30	5

	$K_0^*(800) + K^*(892) + K_0^*(1430)$	
	solution 1	solution 2
$M_{K^*(892)^-}$, MeV$/c^2$	895.42 ± 0.19	895.50 ± 0.22
$\Gamma_{K^*(892)^-}$, MeV	46.14 ± 0.55	46.20 ± 0.69
$\|\gamma\|$	0.954 ± 0.081	1.92 ± 0.20
$\arg(\gamma)$	0.62 ± 0.34	4.03 ± 0.09
$\varkappa$	1.27 ± 0.22	2.28 ± 0.47
$\chi^2/\text{n.d.f.}$	$86.5/84$	$95.1/84$
$P(\chi^2), \%$	41	19

Differential decay rate

The differential decay rate is obtained from

$$d\Gamma = \frac{1}{2m_\tau}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)|\mathcal{M}|^2(2\pi)^4\delta^{(4)}(p_\tau - \sum p_f)$$

Integrating out some angles and momentum,we get

$$\frac{d^2\Gamma}{ds\,d\cos\theta} = \frac{G_F^2\sin^2\theta_C}{2^6\pi^3\sqrt{s}}\frac{(m_\tau^2 - s)^2}{m_\tau^3}P(s)$$
$$\Bigg(\Big(\frac{m_\tau^2}{s}\cos^2\theta + \sin^2\theta\Big)P^2(s)|F_V(s)|^2$$
$$+\frac{m_\tau^2}{4}|F_S(s)|^2 - \frac{m_\tau^2}{\sqrt{s}}P^2(s)Re(F_V F_S^*)\cos\theta\Bigg)$$

The differential forward-backward asymmetry

$$A_{\mathrm{FB}}(s) = \frac{\int_0^1 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta - \int_{-1}^0 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta}{\int_0^1 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta + \int_{-1}^0 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta}$$

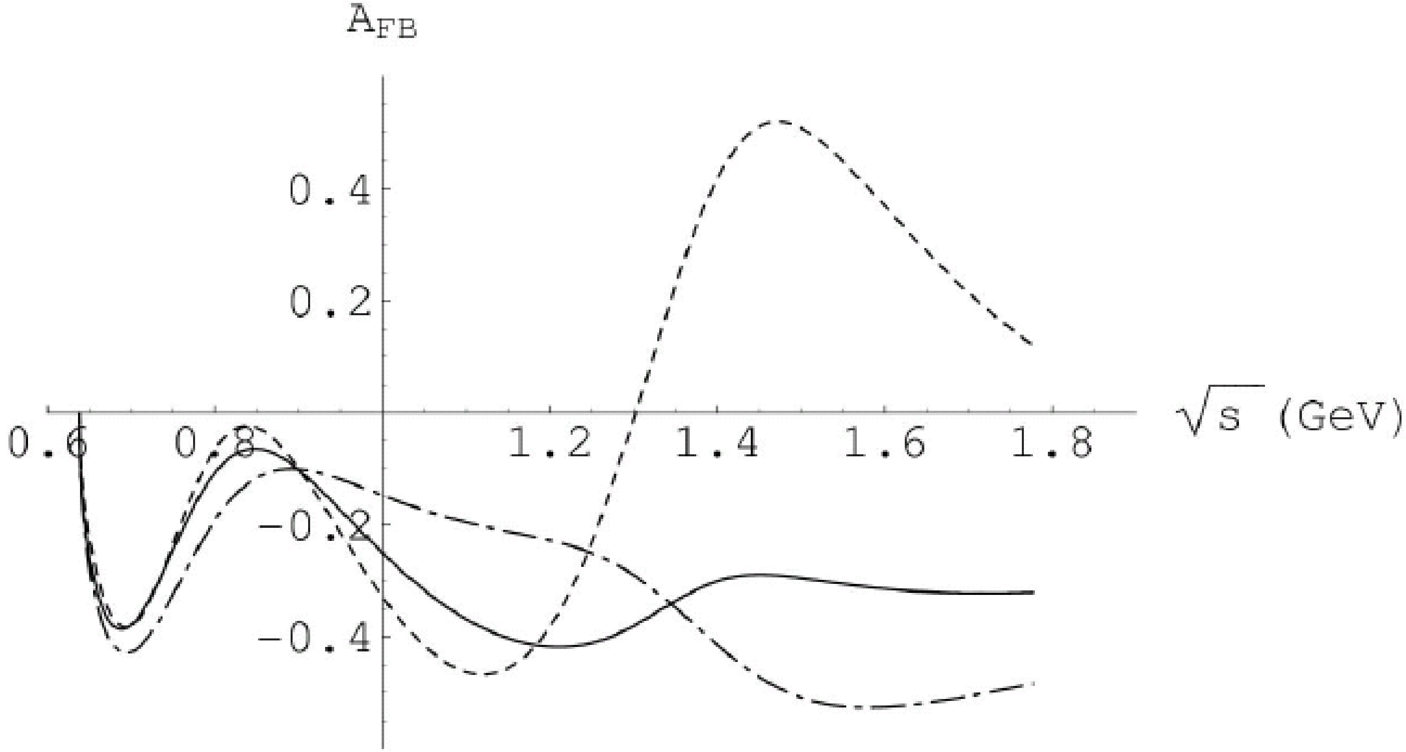


Figure 3: The differential forward-backward asymmetry $A_{\rm FB}$ is plotted as the function of $\sqrt{s}$. The solid line is for Model I, the dashed-dotted line is for Model II-1, and the dashed line is for Model II-2.