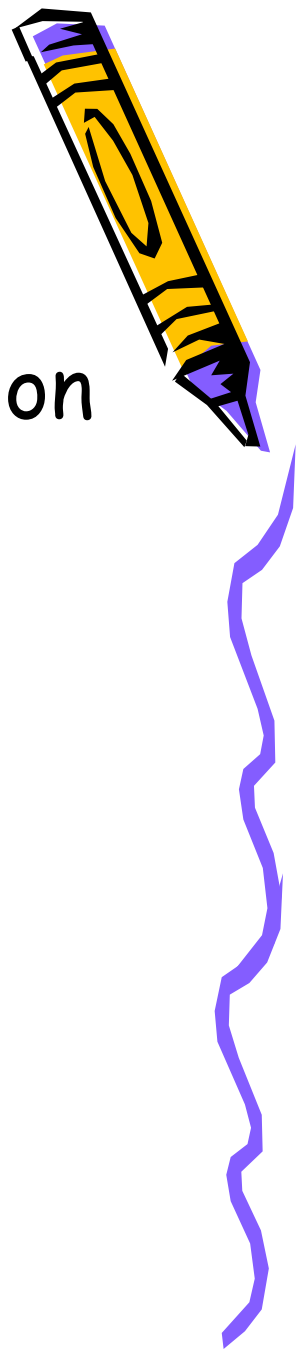


Perturbative QCD in Tau-Charm Energy Region



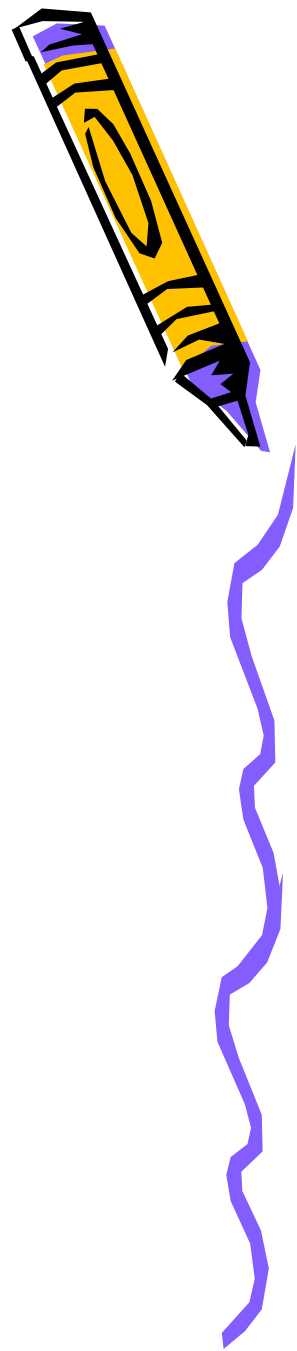
J.P. Ma, ITP, Beijing

科技大, 2013.6.16.



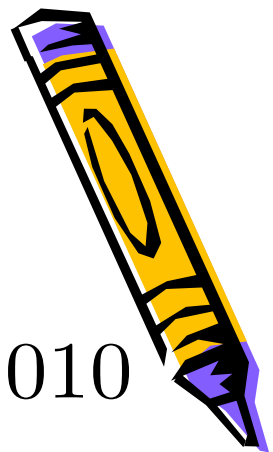
Outline:

1. Inclusive Decay of Tau
2. $e^+e^- \rightarrow \text{hadrons}$
3. NRQCD and Charmonia
4. Outlook



1. Inclusive Decay of Tau

$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{Hadrons})}{\Gamma(\tau \rightarrow \nu_\tau + \mu + \bar{\nu}_\mu)} = 3.640 \pm 0.010$$



Determination of $\alpha_s(m_\tau)$

Spectral functions:

$$\int d^4x e^{iq \cdot x} \sum_X \langle 0 | J^\mu(x) | X \rangle \langle X | J^\nu(0) | 0 \rangle = \pi (q_\mu q_\nu - q^2 g^{\mu\nu}) \rho_{V/A}(q^2) + \dots,$$

$$J^\mu = \bar{u} \gamma^\mu d, \quad \bar{u} \gamma^\mu \gamma_5 d,$$

$d \rightarrow s$, strange quark



..... Power-suppressed, like quark masses, pion decay constant

$$R_\tau \sim \sum_{ud,us} \int_0^{m_\tau^2} ds \omega(s) \left[\rho_V(s) + \rho_A(s) \right] + \dots$$

$$\omega(s) = \left(1 - \frac{s}{m_\tau^2} \right)^2 \left(1 + 2 \frac{s}{m_\tau^2} \right),$$

Flavor separation:

$$R_\tau = R_{V,ud} + R_{A,ud} + R_{\tau,s},$$

Large power correction in the part with s-quark

The s-quark part can not be predicted precisely.



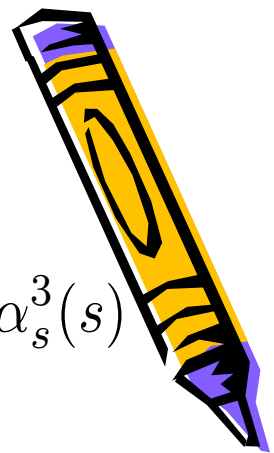
Perturbative QCD:

$$\rho_V(s) = \rho_A(s) = 3 \left[1 + \alpha_s(s) + K_2 \alpha_s^2(s) + \left(K_3 - \frac{1}{3} \pi^2 \beta_0^2 \right) \alpha_s^3(s) \right. \\ \left. + \left(K_4 - \frac{5}{6} - \frac{5}{6} \pi^2 \beta_0 \beta_1 - K_2 \pi^2 \beta_0^2 \right) \alpha_s^4(s) + \dots \right]$$

$$K_2 = 1.9857 - 0.1153 n_f, \quad K_3 = 18.2428 - 4.2158 n_f + 0.862 n_f^2,$$

$$K_4 = 135.7916 - 34.4402 n_f + 1.8753 n_f^2 - 0.0101 n_f^3$$

.. 5-loop!



Theoretical results: (only for ud quark)

Perturbative results + power-correction (nonperturbative effects)

A typical sum-rule problem.....

$$\alpha_s(m_\tau^2) = 0.325 \pm 0.018 \quad (\overline{MS}, n_f = 3, \text{FOPT}) ,$$

$$\alpha_s(m_\tau^2) = 0.347 \pm 0.025 \quad (\overline{MS}, n_f = 3, \text{CIPT}) ,$$

$$\alpha_s(M_Z^2) = 0.1191 \pm 0.0022 \quad (\overline{MS}, n_f = 5, \text{FOPT}) ,$$

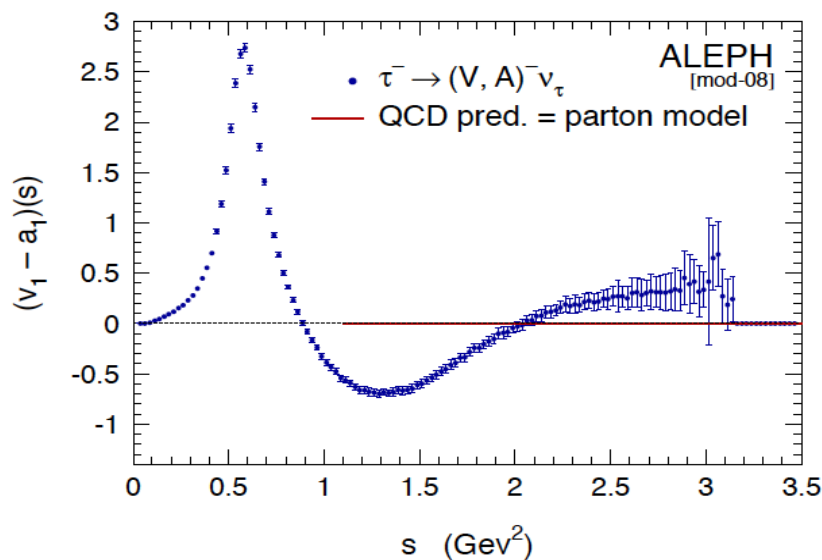
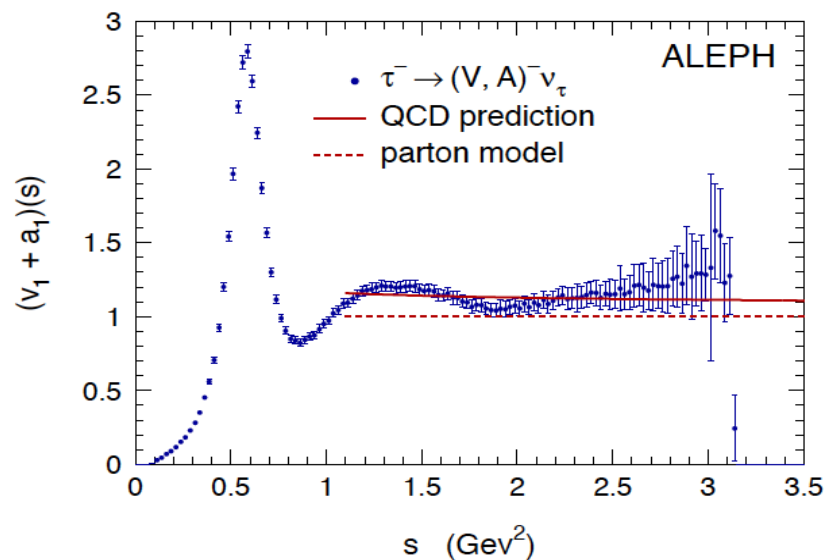
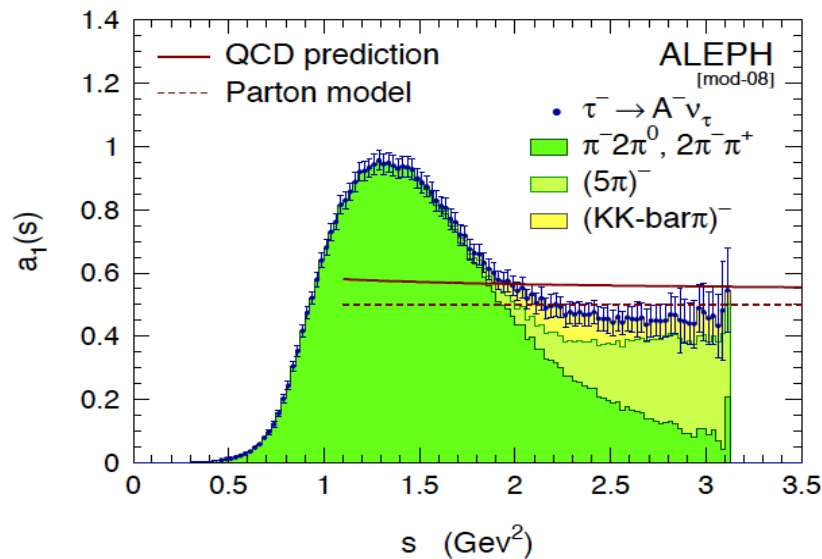
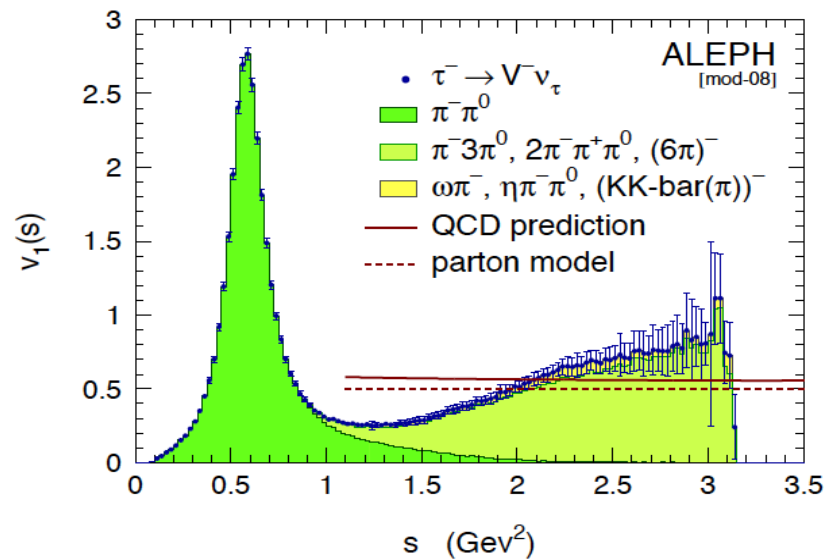
$$\alpha_s(M_Z^2) = 0.1216 \pm 0.0027 \quad (\overline{MS}, n_f = 5, \text{CIPT}) ,$$

Ref: Boito, et. al. ,
Beneke, Boito, Jamin,
Davier, et. al.,

arXiv: 1203.3146,
arXiv: 1210.8238,
arXiv: 0803.0979

Exp. Data: ALEPH 2008





Exp. Data:

$$R_{\tau,s} = 0.1615 \pm 0.0040$$

From kaon....

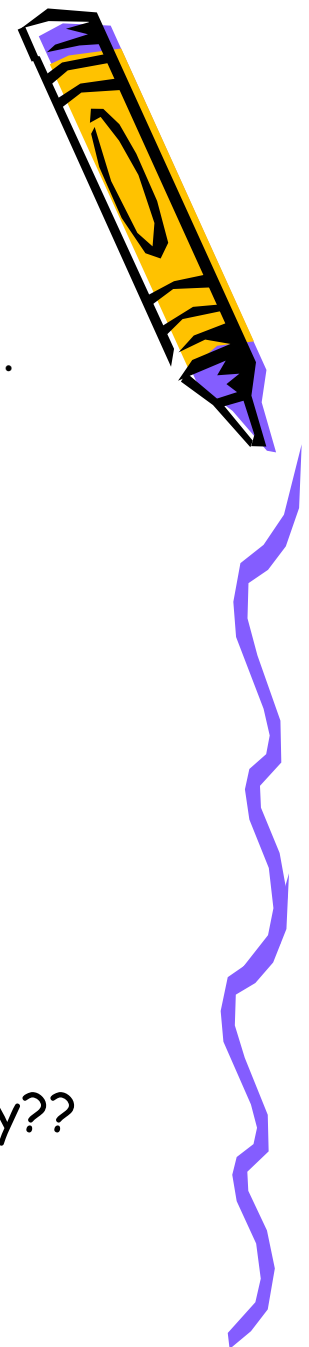
$$R_{\tau,V} = 1.783 \pm 0.011 \pm 0.002 ,$$

$$R_{\tau,A} = 1.695 \pm 0.011 \pm 0.002 ,$$

$$R_{\tau,V+A} = 3.479 \pm 0.011 ,$$

$$R_{\tau,V-A} = 0.087 \pm 0.018 \pm 0.003 ,$$

Improvement by Super Tau-Charm Factory??



2. $e^+e^- \rightarrow \text{hadrons}$

2.1 Inclusive Processes

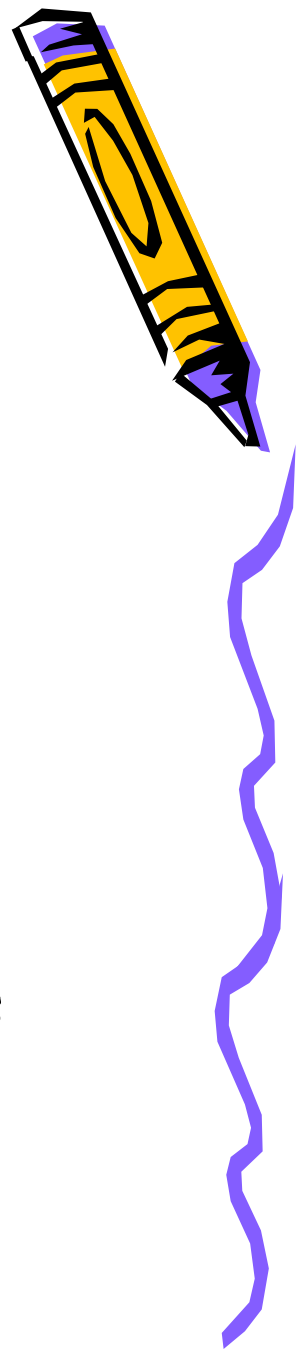
$$R(s) = \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

Perturbative QCD:

$$R(s) = \sum_i Q_i^2 \rho_V(s) + \dots$$

Power-suppressed nonperturbative effects

can be estimated....

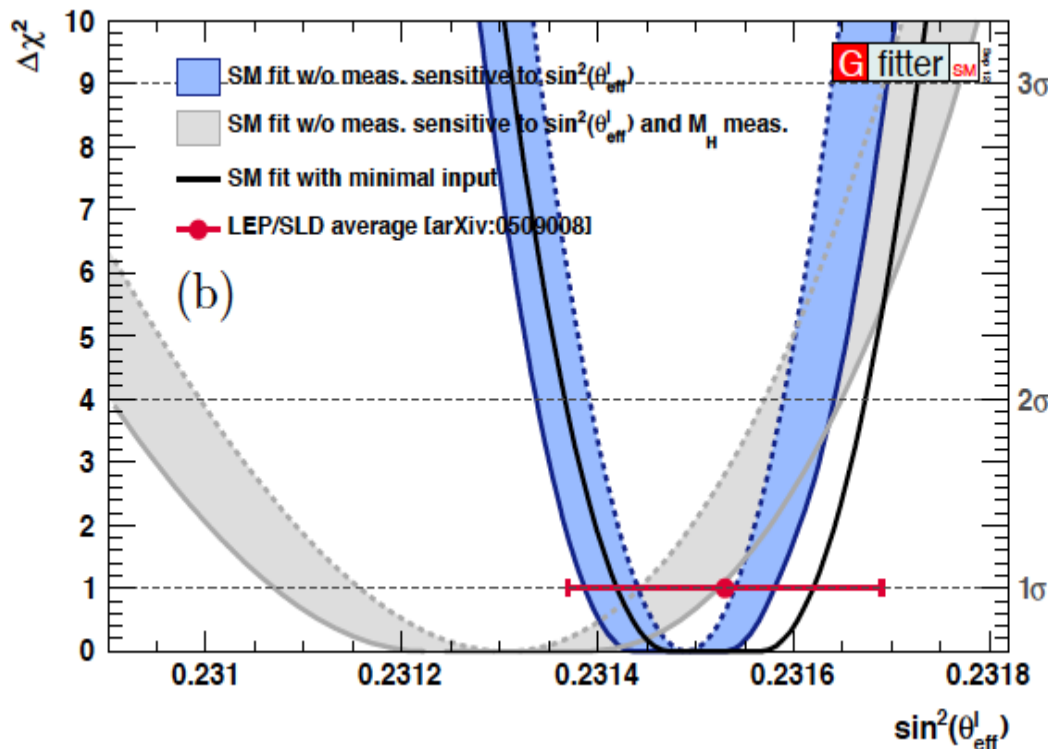


It has been important for precision test of electro-weak sector.

E.g., R gives an important part to determine the running of electroweak coupling. It gave before 07. 2012:

$$M_H = 96^{+31}_{-24} \text{ GeV!}$$

With the discovery of Higgs, the precision is improved !! E.g.,



Gfitter Group: arXiv:1306.571

It is still an important quantity!

E.g., useful for $g-2$.

2.2 Semi Inclusive Processes

Single hadron $e^+e^- \rightarrow h + X$

QCD collinear factorization:

$$\begin{aligned} \frac{d\sigma(e^+e^- \rightarrow h + X)}{dz} &= \sum_{a=q,\bar{q},g} \int \frac{d\xi}{\xi} H_a\left(\frac{z}{\xi}, Q^2, \mu^2\right) D_{a \rightarrow h}(\xi, \mu^2) \\ &= \sum_q \sigma(e^+e^- \rightarrow q\bar{q}) \left(D_{q \rightarrow h}(z) + D_{\bar{q} \rightarrow h}(z) \right) + \mathcal{O}(\alpha_s), \end{aligned}$$

z : Energy fraction of h .

The perturbative coefficient functions are known at one-loop or even at two-loop (?)

D 's: Fragmentation functions.

Extracting FF ???



Two hadrons in final state:

$$e^+ e^- \rightarrow h_1 + h_2 + X$$

🍏 Di-hadron fragmentation function: Important for RHIC, QGP...
but, here the energy is too small to do it

🍏 Collins effect: Each hadron in a jet, two back-to-back jets

A correlation in fragmentation:

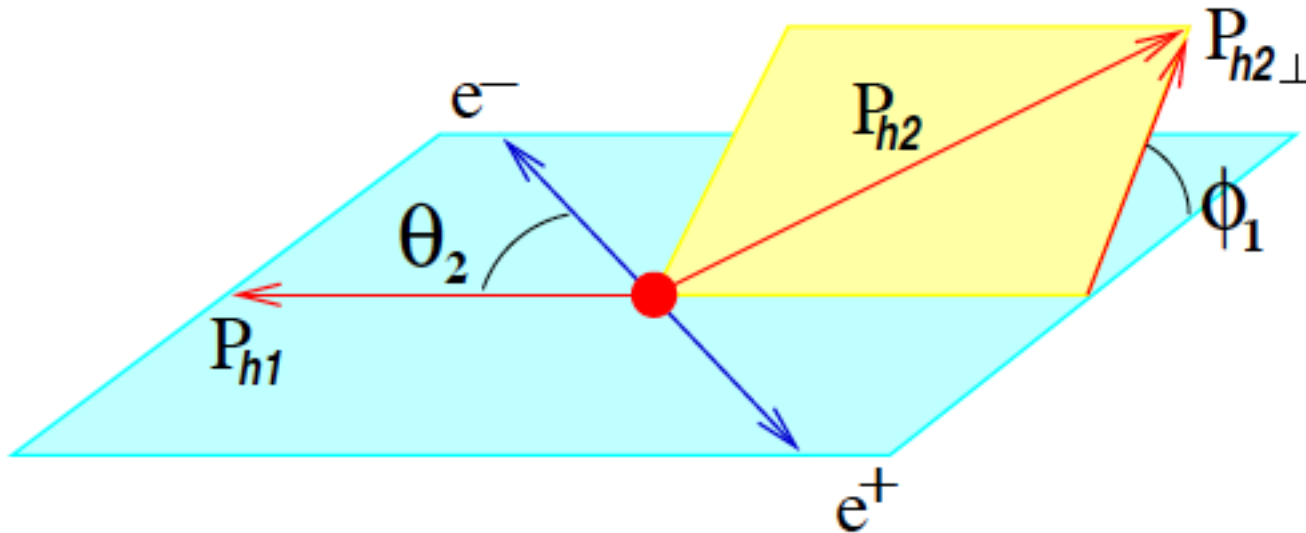
$$q(k, \vec{s}_\perp) \rightarrow h_2(xk + p_\perp) + X$$

A correlation between two transverse vectors.

Collins's FF in QCD TMD factorization.
Breaking of chiral symmetry!



There is an azimuthal angle dependence !!



Belle has observed such a dependence !!

Important: *Sivers vs Collins in SIDIS!!*

More than two hadrons ??

2.3 Exclusive Processes

Two hadrons: form factors

Perturbative QCD: Only one-loop calculation for pion form factor in collinear factorization. (Universal for all mesons at leading twist)

Baryon + Anti-Baryon: e.g., proton form factor

Enhancement near the threshold observed by Babar,

another $X(1835)$????



3. NRQCD and Charmonia

Many processes involving one charmonium or two can be studied with perturbative QCD in the framework of NRQCD factorization

In general, any hadron is a superposition of parton states,

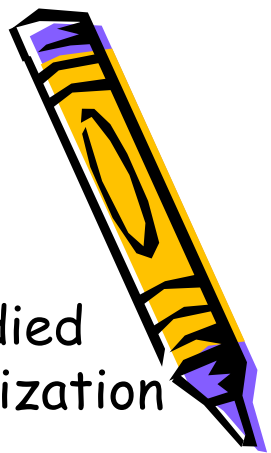
$$|H_{Q\bar{Q}}\rangle = c_0|Q\bar{Q}\rangle + c_1|Q\bar{Q}g\rangle + c_2|Q\bar{Q}gg\rangle + c_3|Q\bar{Q}q\bar{q}\rangle + \dots,$$

For a quarkonium, one knows relative orders of those coefficients, the velocity scaling,

The velocity of the heavy quark: $v \ll 1$

An expansion in $v \rightarrow$ NRQCD, NRQCD factorization
 $\rightarrow$ pNRQCD

Well-defined quantum field theory with
d-dim regularization and $\overline{MS}$ schema



For S-wave quarkonium, the dominant component is $|Q\bar{Q}\rangle$

One can calculate potentials, mass-splitting and masses of quarkonium with perturbative QCD

E.g., at leading order for 1S states:
$$M_\Upsilon - M_{\eta_b} = \frac{C_F^4 \alpha_s^4 m_b}{3}$$

But, these calculations of spectrum may not be used for charmonia...

Perturbative QCD for Charmonia →



Inclusive decays: (Or Electromagnetic decays)

QCD factorization with NRQCD:

$$\Gamma(H_{Q\bar{Q}} \rightarrow \text{Hadrons}) = \sum_i \langle O_i \rangle h_i$$

h_i : A perturbative expansion in the strong coupling, known....

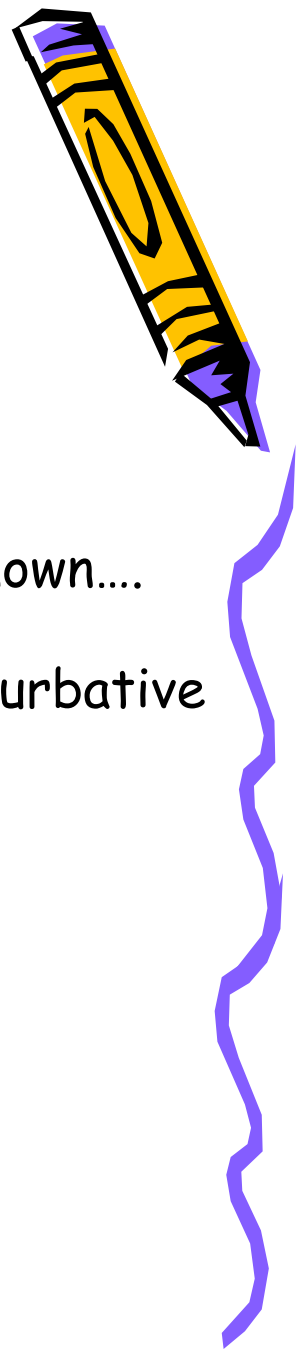
$\langle O_i \rangle$: Well-defined NRQCD matrix elements for nonperturbative effects, unknown.... But,

$$\langle O_i \rangle \propto v^{n_i} \quad \text{Velocity scaling}$$

A double expansion

The factorization is well-established.

Prediction power ?



E.g.,

$$\Gamma(J/\psi \rightarrow l^+ l^-) = \langle O(^3S_1) \rangle \left[a_0 + a_1 \alpha_s + a_2 \alpha_s^2 + \mathcal{O}(\alpha_s^3) \right] \left\{ 1 + \mathcal{O}(v^2) \right\},$$

$$\Gamma(J/\psi \rightarrow X) = \langle O(^3S_1) \rangle \left[b_0 \alpha_s^3 + b_1 \alpha_s^4 + \mathcal{O}(\alpha_s^5) \right] \left\{ 1 + \mathcal{O}(v^2) \right\}$$

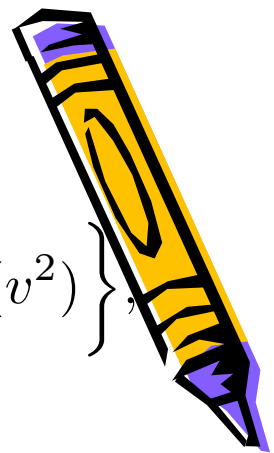
$$\Gamma(J/\psi \rightarrow \gamma + X) = \langle O(^3S_1) \rangle \alpha \left[c_0 \alpha_s^2 + c_1 \alpha_s^3 + \mathcal{O}(\alpha_s^4) \right] \left\{ 1 + \mathcal{O}(v^2) \right\}$$

From one experimental result one can predict other two and more....

Approximated universality of NRQCD matrix elements

→ Prediction power

🍏 The perturbative expansion here is not bad with fewer exception....



For S- and P-wave:

one-loop + relativistic correction are known.

D-wave:

One-loop corrections are known!

Especially: The inclusive decay $\Gamma(\psi(3770) \rightarrow \text{non } D\bar{D})$

Remark: 70% of results obtained by K.T. Chao's group.

There are many theoretical predictions or results for decays to be verified or confirmed by experiment at



Inclusive production:

$$e^+e^- \rightarrow H_Q\bar{Q} + X$$

Most works have been done by K.T. Chao.... J.X. Wang...

An interesting observation by Belle:

$$\frac{\sigma(J/\psi + X_{c\bar{c}})}{\sigma(J/\psi + X_{non\ c\bar{c}})} \sim 1$$



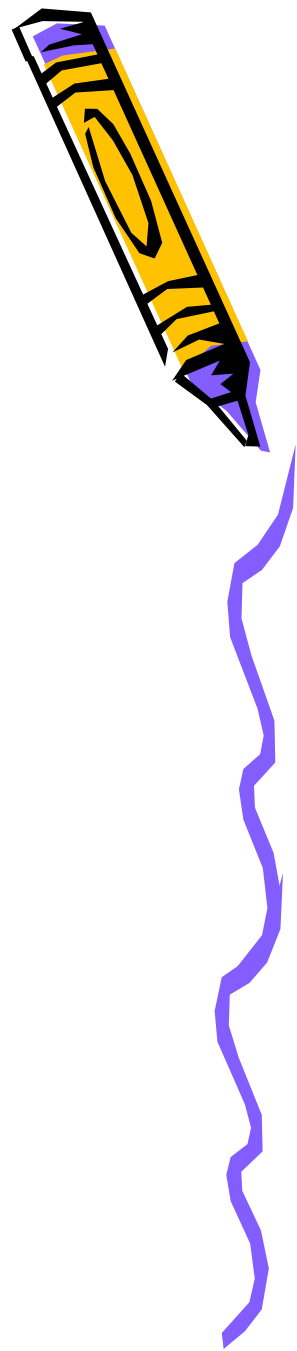
Can be measured at Super tau-charm



$$\sqrt{s} > 6\text{GeV}$$

$$e^+ + e^- \rightarrow H_{c\bar{c}} + H'_{c\bar{c}}$$

Works: K.T. Chao..., J.X. Wang... Y. Jia.....



4. Outlook

Super Tau-Charm Factory ?

