2.1.7.4 $\psi \to \phi f_0$

因为$V \to V'S$过程宇称满足$\eta_a\eta_b\eta_c = 1$，所以末态两个粒子间的相对轨道角动量只能为偶数[$\eta_a\eta_b\eta_c(-1)^l = 1$]；而由于角动量守恒，故$l = 0, 2$。所以末态例子系统只能有两个独立的分波。当宇称守恒时，Helicity 振幅必须满足：

$$F^J_{\lambda\nu} = \eta_a\eta_b\eta_c(-1)^{J_a-J_b-J_c}F^J_{-\lambda-\nu} \tag{2.87}$$

故仅有的两个独立螺旋度振幅如下所示：

$$A = F_1 = F^1_{10} = F^1_{-10}, \qquad B = F_0 = F^1_{00} \tag{2.88}$$

于是可以计算末态角分布（假设$M = +1$）：

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \sum_\lambda |F^1_\lambda D^1_{1,\lambda}(\phi\theta)|^2 = \sum_\lambda |F^1_\lambda \left[d^1_{1,\lambda}(\theta)\right]|^2 \\ &= |A|^2\left\{[d^1_{1,1}(\theta)]^2 + [d^1_{1,-1}(\theta)]^2\right\} + |B|^2[d^1_{1,0}(\theta)]^2 \\ &\propto |A|^2\left[1+\cos^2\theta\right] + |B|^2\sin^2\theta\end{aligned} \tag{2.89}$$

其中A，B 为独立的参数，他们是分波振幅G_{01}，G_{21} 的线性组合，但是由于势垒因子的原因，一般预期D 分量可能要比S 波小很多，则可以假设A 和B 比较接近，这样末态角分布将是近似的均匀分布。这个问题在后面的Helicity振幅的讨论中还会再次提到。如果末态矢量介子ϕ是一个光子，则Helicity振幅F^1_{00}将等于零，末态角分布变成简单的$(1+\cos^2\theta)$。

3.3.3 $\psi \to VA(\phi f_1(1420))$

因为初末态粒子的宇称乘积 $\eta_\psi \eta_\phi \eta_f = +1$，再结合角动量守恒，我们知道独立的分波振幅有三个：G_{01}、G_{21} 和 G_{22}。同样独立Helicity的振幅也只有三个：$F^1_{++} = -F^1_{--}$ 、$F^1_{10} = -F^1_{-10}$ 和 $F^1_{01} = -F^1_{0-1}$ 。

这个过程的两个末态粒子的自旋都是 1 ，宇称相反，它们需要先耦合起来成为 $\chi^{(S)}_{\alpha\beta}$ ，然后再和轨道角动量波函数以及母粒子波函数藕荷而构成振幅，而因为$J+s+\sigma+l=3,5$，所以振幅中有母粒子的四动量出现。如果严格遵从$L-S$ 耦合的概念的指导，则结果是唯一确定的：

$$A_{01} = [p\chi^{(1)}(\lambda\sigma)\phi^*(\delta)] \tag{3.40a}$$

$$A_{21} = [p\chi^{(1)}(\lambda\sigma)\tilde{\tau}^2 \cdot \phi^*(\delta)] \tag{3.40b}$$

$$A_{22} = [p\chi^{(2)}(\lambda\sigma)\tilde{\tau}^2 \cdot \phi^*(\delta)] \tag{3.40c}$$

其中的方括号表示用四阶完全反对称张量进行收缩。最后的化简结果如下：

$$F^1_{1,1} = +g_{01}\sqrt{\frac{1}{6}}r^0 - g_{21}\sqrt{\frac{1}{3}}r^2 \tag{3.41a}$$

$$F^1_{1,0} = +g_{01}\sqrt{\frac{1}{6}}r^0\gamma_\sigma + g_{21}\sqrt{\frac{1}{12}}r^2\gamma_\sigma + g_{22}\sqrt{\frac{1}{4}}r^2\gamma_\sigma \tag{3.41b}$$

$$F^1_{0,1} = +g_{01}\sqrt{\frac{1}{6}}r^0\gamma_s + g_{21}\sqrt{\frac{1}{12}}r^2\gamma_s - g_{22}\sqrt{\frac{1}{4}}r^2\gamma_s \tag{3.41c}$$

2.1.7.5 $\psi \to \phi f_2$

因为$V \to V'T$过程宇称满足$\eta_a\eta_b\eta_c = 1$，所以末态两个粒子间的相对轨道角动量只能为偶数$[\eta_a\eta_b\eta_c(-1)^l = 1]$；而由于角动量守恒，故$l = 0, 2, 4$，总共有五个分波振幅$G_{01}$，$G_{21}$，$G_{22}$，$G_{23}$ 和 G_{43}。所有可能的Helicity振幅如下：

$$F^1_{\lambda\nu} = \begin{pmatrix} F^1_{2\,1} & F^1_{1\,1} & F^1_{0\,1} & F^1_{-1\,1} & F^1_{-2\,1} \\ F^1_{2\,0} & F^1_{1\,0} & F^1_{0\,0} & F^1_{-1\,0} & F^1_{-2\,0} \\ F^1_{2-1} & F^1_{1-1} & F^1_{0-1} & F^1_{-1-1} & F^1_{-2-1} \end{pmatrix},$$

因为角动量守恒，即$|\lambda - \nu| \le J$，所以有$F^1_{-1\,1}$，$F^1_{-2\,1}$，F^1_{2-1}，F^1_{0-1}，$F^1_{2\,0}$和$F^1_{-2\,0}$为零；又根据宇称守恒的关系2.78，仅有的五个独立螺旋度振幅表示如下：

$$\begin{aligned} &F^1_{21} = F^1_{-2-1}, \quad F^1_{11} = F^1_{-1-1}, \quad F^1_{01} = F^1_{0-1}, \\ &F^1_{10} = F^1_{-10} \quad \text{和} \quad F^1_{00}, \end{aligned}$$

现在我们来计算末态角分布（假设$M = +1$）：

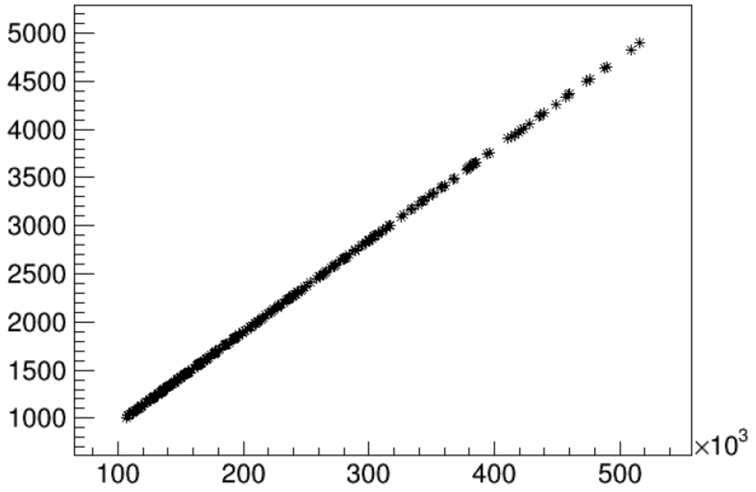
$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \sum_{\lambda\nu} |F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta)|^2 = \sum_{\lambda\nu} |F^1_{\lambda\nu}\left[d^1_{1,\lambda-\nu}(\theta)\right]|^2 \\ &= |F^1_{21}|^2\left\{[d^1_{1,1}(\theta)]^2 + [d^1_{1,-1}(\theta)]^2\right\} \\ &\quad \underline{+2|F^1_{11}|^2[d^1_{1,0}(\theta)]^2} \\ &\quad +|F^1_{01}|^2\left\{[d^1_{1,-1}(\theta)]^2 + [d^1_{1,1}(\theta)]^2\right\} \\ &\quad +|F^1_{10}|^2\left\{[d^1_{1,1}(\theta)]^2 + [d^1_{1,-1}(\theta)]^2\right\} \\ &\quad +|F^1_{00}|^2[d^1_{1,0}(\theta)]^2 \\ &= \frac{1+\cos^2\theta}{2}\left[|F^1_{21}|^2 + |F^1_{10}|^2 + |F^1_{01}|^2\right] \\ &\quad + \frac{\sin^2\theta}{2}\left[2|F^1_{11}|^2 + |F^1_{00}|^2\right] \end{aligned} \tag{2.90}$$

可以看到这个角分布中含有更多的位置参数有待确定。

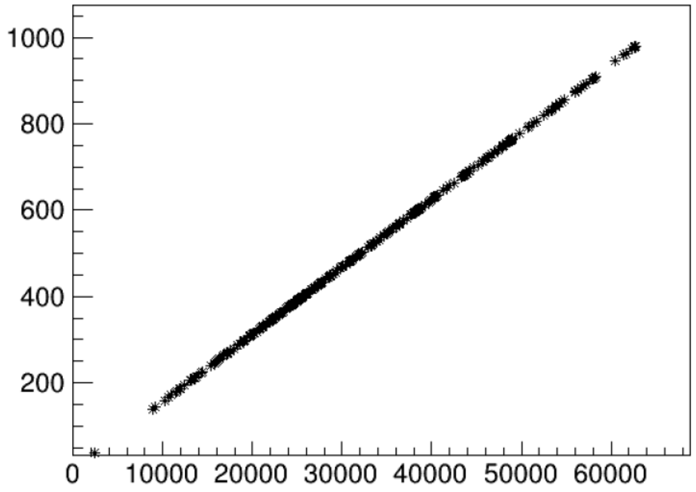
```
  for(int m_MY=-1;m_MY<=1;m_MY+=2)//two values for Y-state helicity
  {
    for(int m_MJ=-1; m_MJ<=1; m_MJ++)
    {
      TComplex amp1_x(0.0,0.0);//g01
      TComplex amp2_x(0.0,0.0);//g21
      double RF_jpsi = RFgamma(1, m_MJ, gamma_jpsi);//Jpsi in Y rest frame
      //g01
      amp1_x = RF_jpsi * Ffunc(1,0,m_MJ,0,1,0,1) * wignerDC_1(1,m_MY,-m_MJ,thetaX,phiX);
      //g21
      amp2_x = RF_jpsi * Ffunc(1,0,m_MJ,0,1,2,1) * wignerDC_1(1,m_MY,-m_MJ,thetaX,phiX)*BarrierF(2,rhoX*2.0);

      TComplex ppXS_g01 = amp1_x;
      TComplex ppXS_g21 = amp2_x;

      amp_terms["f0500_SS"].push_back(CS0_1 * ppXS_g01*tfS0);
      amp_terms["f0500_DS"].push_back(CS0_2 * ppXS_g21*tfS0);
      amp_terms["f0980_SS"].push_back(CS1_1 * ppXS_g01*tfS1);
      amp_terms["f0980_DS"].push_back(CS1_2 * ppXS_g21*tfS1);
      amp_terms["f01370_SS"].push_back(CS2_1 * ppXS_g01*tfS2);
      amp_terms["f01370_DS"].push_back(CS2_2 * ppXS_g21*tfS2);

      //amp_terms["f0500_SS"].push_back(CS0_1 * ppXS_g01);
      //amp_terms["f0500_DS"].push_back(CS0_2 * ppXS_g21);
      //amp_terms["f0980_SS"].push_back(CS1_1 * ppXS_g01);
      //amp_terms["f0980_DS"].push_back(CS1_2 * ppXS_g21);
      //amp_terms["f01370_SS"].push_back(CS2_1 * ppXS_g01);
      //amp_terms["f01370_DS"].push_back(CS2_2 * ppXS_g21);
    }
  }
}
```

```
for(int m_MY=-1;m_MY<=1;m_MY+=2)//two values for Y-state helicity
{
  for(int m_MJ=-1; m_MJ<=1; m_MJ++)
  {
    TComplex amp1_x(0.0,0.0);
    for(int m_Mf=-2; m_Mf<=2; m_Mf++)
    {
      double RF_jpsi = RFgamma(1, m_MJ, gamma_jpsi);//Jpsi in Y rest frame
      double RF_x = RFgamma(2, m_Mf, gamma_x);
      //g_01
      amp1_x += RF_jpsi*RF_x * Ffunc(1,m_Mf,m_MJ,2,1,0,1) * wignerDC_1(1,m_MY,m_Mf-m_MJ,thetaX,phiX) * Ffunc(2,0,0,0,0,2,0) * wignerDC_1(2,m_Mf,0,thetapip,phipip) * BarrierF
    }
    TComplex ppXD_g01 = amp1_x;
    //f21270
    TComplex sum_XD0  = CD0_1 * ppXD_g01;
    amp_terms["f21270"].push_back(sum_XD0*tfD0);
    //amp_terms["f21270"].push_back(sum_XD0);
  }
}
}
```

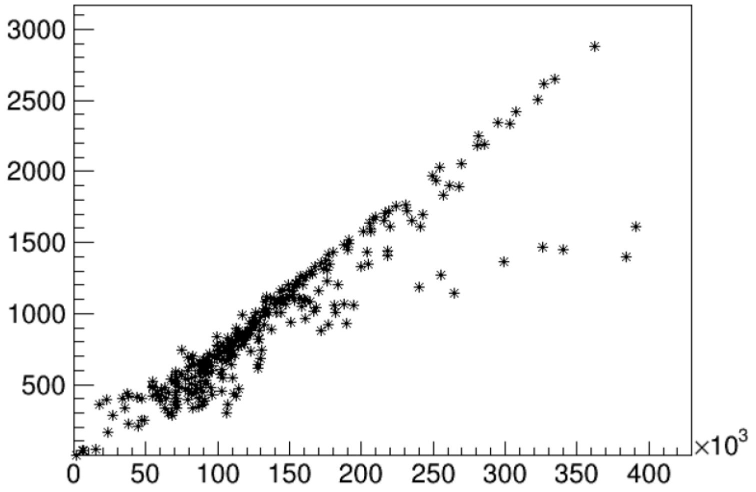
F0 SS



F0 DS



F2 SD



Jpsi is considered as a final state, jpsi->ll is not included in both helicity and covariant tensor formalisms

▼ Decay sequence $\psi \to \phi f_0, f_0 \to \pi^+\pi^-$:

$$\begin{array}{rlllll} Decay: \psi & \to\phi & f_0,f_0 & \to\pi^+ & \pi^- \\ J^{PC}: 1^{--} & \to 1^{--} & 0^+,0^+ & \to 0^+ & 0^+ \end{array}$$

- 由宇称守恒和角动量守恒:
  - 子过程 $\psi \to \phi f_0$ 的可能轨道角动量取值为 $L = 0, 2$
  - 子过程 $f_0 \to \pi^+\pi^-$ 的可能轨道角动量取值为 $l = 0$
- 由此构造振幅:
  - $L = 0$ 时

$$\mathcal{M}_1 = \xi^*_\mu(m_Y)\phi^\mu(m_\phi)BW(f_0)$$

- $L = 2$ 时

$$\mathcal{M}_2 = \xi^*_\mu(m_Y)\phi_\nu(m_\phi)\tilde{T}^{(2)\mu\nu}_{(\phi f_0)}BW(f_0)$$

- 其中, $\xi^*_\mu(m_Y)$是 Y-state 的极化矢量, $\phi^\nu(m_\phi)$是 ϕ 的极化矢量, $\tilde{T}^{(2)\mu\nu}_{(\phi f_0)}$) 是 ϕf_0间轨道角动量为2时贡献的轨道张量

$$
\begin{aligned}
Decay: Y &\rightarrow \psi \quad f_2 \\
J^{PC}: 1^{--} &\rightarrow 1^{--} \quad 2^{+}
\end{aligned}
$$

- **最低阶的 $L=0, S=1$ 的情形:**

1. 先考虑 $s-s$ 耦合 $S \rightarrow s_{f_2} + s_{\psi} \Rightarrow 1 \rightarrow 1+2$: 末态系统的总自旋为1, 系统的总自旋极化矢量为S_{σ}^{*}
   - $s-s$ 耦合中, $S + s_{f_2} + s_{\psi} = 1+1+2 = even$, 无全反对称张量的贡献
   - $s-s$ 耦合振幅为 $\mathcal{M}^{s} = S_{\sigma}^{*}\phi^{\sigma\sigma'}(m_{f_2})\omega_{\sigma'}(m_{\psi})$
2. 再考虑$L-S$耦合, $J \rightarrow S+L \Rightarrow 1 \rightarrow 1+0$: ψ和f_2形成的系统的总角动量为J
   - $J+S+L = 1+1+0 = even$, 无全反对称张量的贡献
   - $L=0$, 无轨道角动量贡献

- 此子过程的振幅为: $\mathcal{M}^{L} = \xi_{\mu}^{*}(m_Y)U^{\mu}, U^{\mu} = S^{\mu}$

体系的$L-S$耦合振幅为:

$$
\begin{aligned}
\mathcal{M}^{LS} &= \xi_{\mu}^{*}(m_Y)S^{\mu}S_{\sigma}^{*}\phi^{\sigma\sigma'}(m_{f_2})\omega_{\sigma'}(m_{\psi}) \\
&= \xi_{\mu}^{*}(m_Y)P_{\sigma}^{(1)\mu}(p_Y)\phi^{\sigma\sigma'}(m_{f_2})\omega_{\sigma'}(m_{\psi}) \\
&= \xi_{\sigma}^{*}(m_Y)\phi^{\sigma\sigma'}(m_{f_2})\omega_{\sigma'}(m_{\psi})
\end{aligned}
$$

不考虑$f_2 \rightarrow \pi^{+}\pi^{-}$这一级

微分截面仅为第一级的模方:

$$
\begin{aligned}
\frac{d\sigma}{d\Phi_n} &\propto \frac{1}{2}\sum_{m_1}^{2}\sum_{m_2}^{3}\sum_{m_3}^{5}\xi_{\mu}^{*}(m_1)\omega_{\nu}(m_2)\phi(m_3)^{\mu\nu}\xi_{\mu'}(m_1)\omega_{\nu'}^{*}(m_2)\phi(m_3)^{*\mu'\nu'} \\
&= -\frac{1}{2}\sum_{\mu=1}^{2}\Lambda\Lambda^{*}\cdot\tilde{g}_{\nu\nu'}(p_{(\psi)})P^{(2)\mu\nu\mu\nu'}(p_{(f2)}) \\
&= -\frac{1}{2}\sum_{\mu=1}^{2}\Lambda\Lambda^{*}\cdot\tilde{g}^{\nu\nu'}(p_{(\psi)})P_{\mu\nu\mu\nu'}^{(2)}(p_{(f2)})
\end{aligned}
$$

Helicity formalism

$$F^1_{\lambda\nu} = \begin{pmatrix} F^1_{21} & F^1_{11} & F^1_{01} & F^1_{-11} & F^1_{-21} \\ F^1_{20} & F^1_{10} & F^1_{00} & F^1_{-10} & F^1_{-20} \\ F^1_{2-1} & F^1_{1-1} & F^1_{0-1} & F^1_{-1-1} & F^1_{-2-1} \end{pmatrix}$$

- 因为角动量守恒要求$|\lambda-\nu| \leq J$，所以$F^1_{1,-1}$，$F^1_{-1,1}, F^1_{-2,1}, F^1_{2,-1}, F^1_{2,0}, F^1_{-2,0} = 0$
- 由宇称守恒关系，仅有5个独立的振幅：

$$F^1_{21} = F^1_{-2-1}, \quad F^1_{11} = F^1_{-1-1}, \quad F^1_{01} = F^1_{0-1}, \quad F^1_{10} = F^1_{-10}, \quad F^1_{00},$$

$$F^J_{\lambda\nu} = \sum_{ls} g_{\ell S} A_{\ell S}(\lambda\nu)$$

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{1/2} (\ell 0 S\delta \mid J\delta)(s\lambda\sigma-\nu \mid S\delta) \times W^n r^\ell f^s_\lambda(\gamma_s)\, f^\sigma_\nu(\gamma_\sigma),$$

$$\begin{aligned}
\sum_{\lambda\nu} \left|F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta)\right|^2 &= \sum_{\lambda\nu} \left|F^1_{\lambda\nu}\left[d^1_{1,\lambda-\nu}(\theta)\right]\right|^2 \\
&= \left|F^1_{21}\right|^2 \left\{\left[d^1_{1,1}(\theta)\right]^2 + \left[d^1_{1,-1}(\theta)\right]^2\right\} \\
&+ 2\left|F^1_{11}\right|^2 \left[d^1_{1,0}(\theta)\right]^2 \\
&+ \left|F^1_{01}\right|^2 \left\{\left[d^1_{1,-1}(\theta)\right]^2 + \left[d^1_{1,1}(\theta)\right]^2\right\} \\
&+ \left|F^1_{10}\right|^2 \left\{\left[d^1_{1,1}(\theta)\right]^2 + \left[d^1_{1,-1}(\theta)\right]^2\right\} \\
&+ \left|F^1_{00}\right|^2 \left[d^1_{1,0}(\theta)\right]^2 \\
&= \frac{1+\cos^2\theta}{2}\left[\left|F^1_{21}\right|^2 + \left|F^1_{10}\right|^2 + \left|F^1_{01}\right|^2\right] \\
&+ \frac{\sin^2\theta}{2}\left[2\left|F^1_{11}\right|^2 + \left|F^1_{00}\right|^2\right]
\end{aligned}$$

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \sum_{M\lambda\nu} \left|F^1_{\lambda\nu} D^1_{M,\lambda-\nu}(\phi\theta)\right|^2 = 2\sum_{\lambda\nu} \left|F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta)\right|^2 = 2\sum_{\lambda\nu} \left|\sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu) d^1_{1,\lambda-\nu}\right|^2 \\
&= 2\left|g_{01}\right|^2 \cdot \left\{\frac{1+\cos^2\theta}{2}\left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2\right] + \frac{\sin^2\theta}{2}\left[|A_{01}(11)|^2 + |A_{01}(00)|^2\right]\right\}
\end{aligned}$$

$$Decay: Y \rightarrow \psi \quad f_2$$
$$J^{PC}: 1^{--} \rightarrow 1^{--} \quad 2^{+}$$