Perturbative QCD in Tau-Charm Energy Region

J.P. Ma, ITP, Beijing

衡阳 **2018.10.13.**

Outline:

1. Inclusive Decay of Tau
2. $e^+e^- \to$ hadrons
3. NRQCD and Charmonia
4. Outlook

1. Inclusive Decay of Tau

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau + \mathrm{Hadrons})}{\Gamma(\tau \to \nu_\tau + \mu + \bar{\nu}_\mu)} = 3.640 \pm 0.010$$

Determination of $\alpha_s(m_\tau)$

Spectral functions:

$$\int d^4 x e^{iq\cdot x} \sum_X \langle 0|J^\mu(x)|X\rangle\langle X|J^\nu(0)|0\rangle = \pi(q_\mu q_\nu - q^2 g^{\mu\nu})\rho_{V/A}(q^2) + \cdots,$$

$$J^\mu = \bar{u}\gamma^\mu d, \quad \bar{u}\gamma^\mu\gamma_5 d,$$

d -> s, strange quark

….. Power-suppressed, like quark masses, pion decay constant

$$R_\tau \sim \sum_{ud,us} \int_0^{m_\tau^2} ds\omega(s)\Big[\rho_V(s) + \rho_A(s)\Big] + \cdots$$

$$\omega(s) = \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right),$$

Flavor separation:

$$R_\tau = R_{V,ud} + R_{A,ud} + R_{\tau,s},$$

Large power correction in the part with s-quark

The s-quark part can not be predicted precisely.

Perturbative QCD:

$$\rho_V(s) = \rho_A(s) = 3\Big[1 + \alpha_s(s) + K_2\alpha_s^2(s) + \left(K_3 - \frac{1}{3}\pi^2\beta_0^2\right)\alpha_s^3(s)$$
$$+\left(K_4 - \frac{5}{6} - \frac{5}{6}\pi^2\beta_0\beta_1 - K_2\pi^2\beta_0^2\right)\alpha_s^4(s) + \cdots$$

$$K_2 = 1.9857 - 0.1153n_f, \quad K_3 = 18.2428 - 4.2158n_f + 0.862n_f^2,$$
$$K_4 = 135.7916 - 34.4402n_f + 1.8753n_f^2 - 0.0101n_f^3$$

.. 5-loop!

Theoretical results: (only for ud quark)

Perturbative results + power-correction (nonperturbative effects)

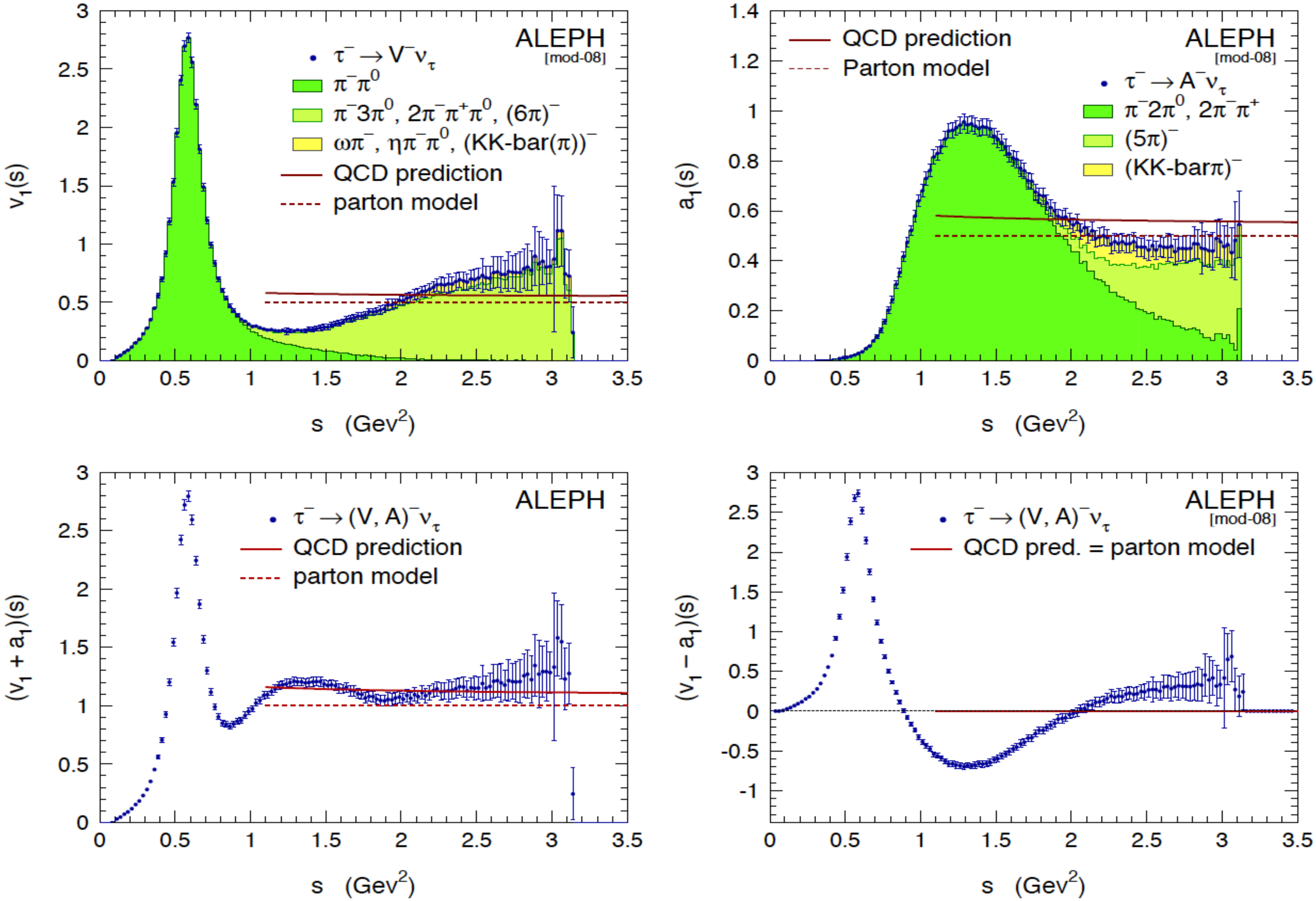
A typical sum-rule problem……

$$\alpha_s(m_\tau^2) = 0.325 \pm 0.018 \qquad (\overline{MS},\ n_f = 3,\ \text{FOPT})\ ,$$
$$\alpha_s(m_\tau^2) = 0.347 \pm 0.025 \qquad (\overline{MS},\ n_f = 3,\ \text{CIPT})\ ,$$

$$\alpha_s(M_Z^2) = 0.1191 \pm 0.0022 \qquad (\overline{MS},\ n_f = 5,\ \text{FOPT})\ ,$$
$$\alpha_s(M_Z^2) = 0.1216 \pm 0.0027 \qquad (\overline{MS},\ n_f = 5,\ \text{CIPT})\ ,$$

Ref: Boito, et. al. , arXiv: 1203.3146,
Beneke, Boito, Jamin, arXiv: 1210.8238,
Davier, et. al., arXiv: 0803.0979

Exp. Data: ALEPH 2008



ALEPH
[mod-08]
τ⁻ → V⁻ν_τ
π⁻π⁰
π⁻3π⁰, 2π⁻π⁺π⁰, (6π)⁻
ωπ⁻, ηπ⁻π⁰, (KK-bar(π))⁻
QCD prediction
parton model
v_1(s)
s (Gev²)
ALEPH
[mod-08]
QCD prediction
Parton model
τ⁻ → A⁻ν_τ
π⁻2π⁰, 2π⁻π⁺
(5π)⁻
(KK-barπ)⁻
a_1(s)
s (Gev²)
ALEPH
τ⁻ → (V, A)⁻ν_τ
QCD prediction
parton model
(v_1 + a_1)(s)
s (Gev²)
ALEPH
[mod-08]
τ⁻ → (V, A)⁻ν_τ
QCD pred. = parton model
(v_1 − a_1)(s)
s (Gev²)

Exp. Data:

$$R_{\tau,s} = 0.1615 \pm 0.0040$$

From kaon....

$$
\begin{aligned}
R_{\tau,V} &= 1.783 \pm 0.011 \pm 0.002\,, \\
R_{\tau,A} &= 1.695 \pm 0.011 \pm 0.002\,, \\
R_{\tau,V+A} &= 3.479 \pm 0.011\,, \\
R_{\tau,V-A} &= 0.087 \pm 0.018 \pm 0.003\,,
\end{aligned}
$$

Improvement by Super Tau-Charm Factory??

2. $e^+e^- \to$ hadrons

2.1 Inclusive Processes

$$R(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Perturbative QCD:

$$R(s) = \sum_i Q_i^2 \rho_V(s) + \cdots$$

Power-suppressed nonperturbative effects

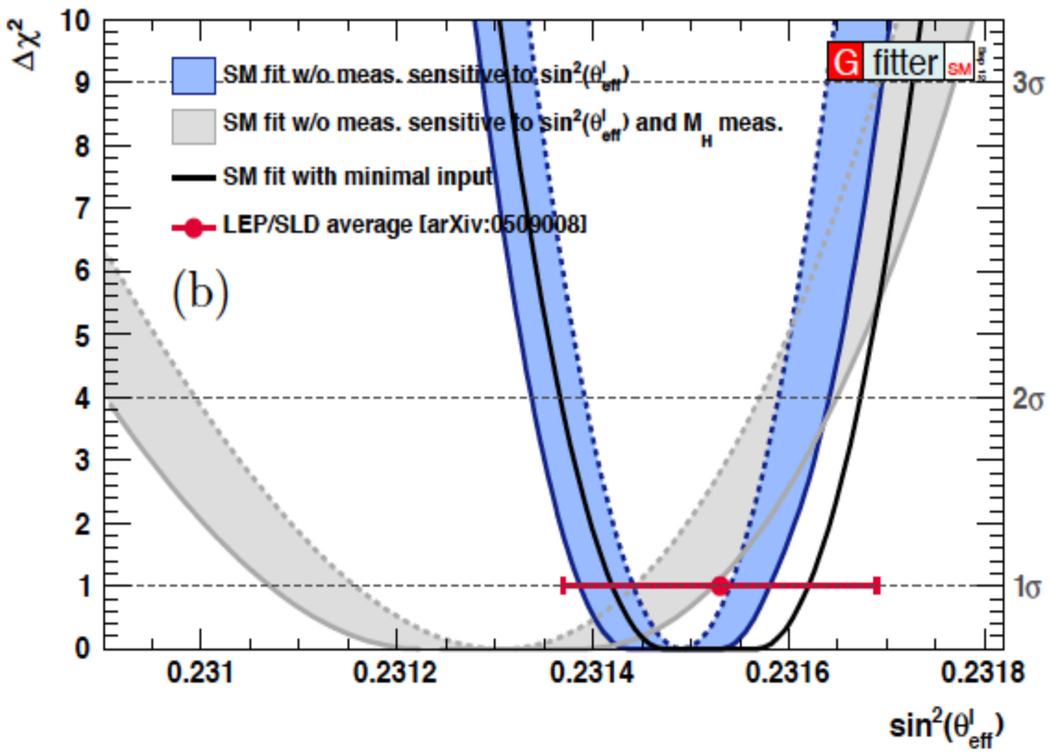
can be estimated....

It has been important for precision test of electro-weak sector!

E.g., R gives an important part to determine the running of electroweak coupling. It gave before 07. 2012:

$$M_H = 96^{+31}_{-24}\mathrm{GeV!}$$

With the discovery of Higgs, the precision is improved !! E.g.,



Gfitter Group: arXiv:1306.571

It is still an important quantity!

E.g., useful for g-2.

2.2 Semi Inclusive Processes

Single hadron $e^+e^- \to h + X$

QCD collinear factorization:

$$\frac{d\sigma(e^+e^- \to h+X)}{dz} = \sum_{a=q,\bar{q},g} \int \frac{d\xi}{\xi} H_a(\frac{z}{\xi}, Q^2, \mu^2) D_{a\to h}(\xi, \mu^2)$$

$$= \sum_q \sigma(e^+e^- \to q\bar{q}) \Big(D_{q\to h}(z) + D_{\bar{q}\to h}(z) \Big) + \mathcal{O}(\alpha_s),$$

z: Energy fraction of h.

The perturbative coefficient functions are known at one-loop or even at two-loop (?)

D's: Fragmentation functions.

Extracting FF ???

Two hadrons in final state:

$$e^+e^- \rightarrow h_1 + h_2 + X$$

- Di-hadron fragmentation function: Important for RHIC, QGP… but, here the energy is too small to do it

- Collins effect: Each hadron in a jet, two back-to- back jets

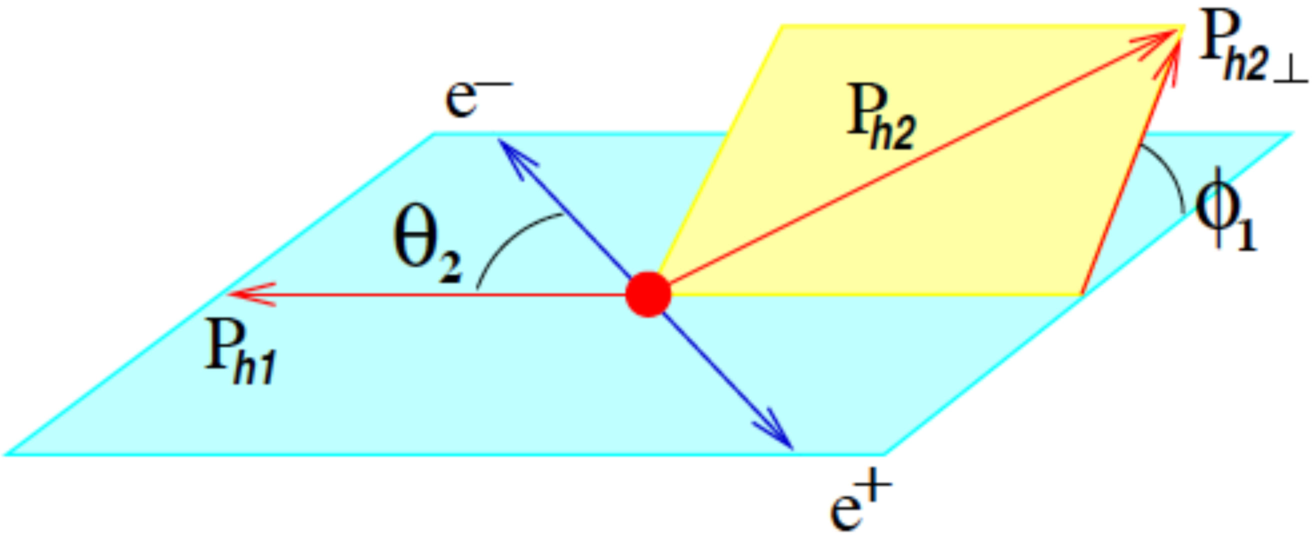
A correlation in fragmentation:

$$q(k, \vec{s}_\perp) \rightarrow h_2(xk + p_\perp) + X$$

A correlation between two transverse vectors.

Collins's FF in QCD TMD factorization.
Breaking of chiral symmetry!

There is an azimuthal angle dependence !!



Belle has observed such a dependence !!

Important: Sivers vs Collins in SIDIS!!

More than two hadrons ??

2.3 Exclusive Processes

Two hadons: form factors

Perturbative QCD: Only one-loop calculation for pion form factor in collinear factorization. (Universal for all mesons at leading twist)

Baryon +Anti-Baryon: e.g., proton form factor

Enhancement near the threshold observed by Babar,

another X(1835) ????

3. NRQCD and Charmonia

Many processes involving one charmonium or two can be studied with perturbative QCD in the framework of NRQCD factorization

In general, any hadron is a superposition of parton states,

$$|H_{Q\bar{Q}}\rangle = c_0|Q\bar{Q}\rangle + c_1|Q\bar{Q}g\rangle + c_2|Q\bar{Q}gg\rangle + c_3|Q\bar{Q}q\bar{q}\rangle + \cdots,$$

For a quarkonium, one knows relative orders of those coefficients, the velocity scaling,

The velocity of the heavy quark: $v \ll 1$

An expansion in v ➔ NRQCD, NRQCD factorization
➔ pNRQCD

Well-defined quantum field theory with d-dim regularization and MS schema

For S-wave quarkonium, the dominant component is $|Q\bar{Q}\rangle$

One can calculate potentials, mass-splitting and masses of quarkonium with perturbative QCD

E.g., at leading order for 1S states: $M_{\Upsilon} - M_{\eta_b} = \frac{C_F^4 \alpha_s^4 m_b}{3}$

But, these calculations of spectrum may not be used for charmonia…

Perturbative QCD for Charmonia ➔

Inclusive decays: (Or Electromagnetic decays)

QCD factorization with NRQCD:

$$\Gamma(H_{Q\bar{Q}} \to \text{Hadrons}) = \sum_i \langle O_i \rangle \; h_i$$

h_i : A perturbative expansion in the strong coupling, known….

$\langle O_i \rangle$: Well-defined NRQCD matrix elements for nonperturbative effects, unknown…. But,

$$\langle O_i \rangle \propto v^{n_i}$$ Velocity scaling

A double expansion

The factorization is well-established.

Prediction power ?

E.g.,

$$\Gamma(J/\psi \to l^+ l^-) = \langle O(^3S_1)\rangle \left[a_0 + a_1\alpha_s + a_2\alpha_s^2 + \mathcal{O}(\alpha_s^3)\right]\left\{1 + \mathcal{O}(v^2)\right\},$$

$$\Gamma(J/\psi \to X) = \langle O(^3S_1)\rangle \left[b_0\alpha_s^3 + b_1\alpha_s^4 + \mathcal{O}(\alpha_s^5)\right]\left\{1 + \mathcal{O}(v^2)\right\}$$

$$\Gamma(J/\psi \to \gamma + X) = \langle O(^3S_1)\rangle \alpha \left[c_0\alpha_s^2 + c_1\alpha_s^3 + \mathcal{O}(\alpha_s^4)\right]\left\{1 + \mathcal{O}(v^2)\right\}$$

From one experimental result one can predict other two and more….

Approximated universality of NRQCD matrix elements
➔ Prediction power

- The perturbative expansion here is not bad with fewer exception….

For S- and P-wave:

one-loop + relativistic correction are known.

D-wave:

One-loop corrections are known!

Especially: The inclusive decay $\Gamma(\psi(3770) \to \text{non } D\bar{D})$

Remark: 70% of results obtained by K.T. Chao's group.

There are many theoretical predictions or results for decays to be verified or confirmed by experiment at .....

Inclusive production:

$$e^+e^- \to H_{Q\bar{Q}} + X$$

Most works have been done by K.T. Chao.... J.X. Wang...

An interesting observation by Belle:

$$\frac{\sigma(J/\psi + X_{c\bar{c}})}{\sigma(J/\psi + X_{non\ c\bar{c}})} \sim 1$$

Can be measured at Super tau-charm

$$\sqrt{s} > 6\mathrm{GeV}$$

$$e^+ + e^- \to H_{c\bar{c}} + H'_{c\bar{c}}$$

Works: K.T. Chao…, J.X. Wang… Y. Jia……

4. Outlook

Super Tau-Charm Factory ?