Lecture for experimental students

Guide to understanding resonance

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Contents

0. Introduction
1. What is resonance ?
2. How to find resonance ?
3. Fake of resonance

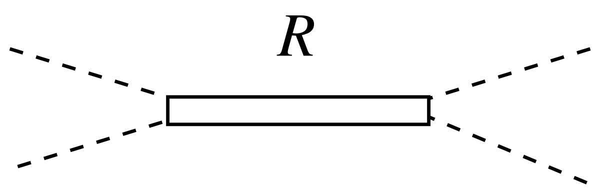
Textbook : John R. Taylor, "Scattering Theory"

Although I did not consult this textbook when preparing this lecture,

many of the contents should be found in this standard textbook

0. Introduction

Quasi-stable intermediate state formed in scattering process → resonance



Common experimentalists' procedure of extracting resonances from data

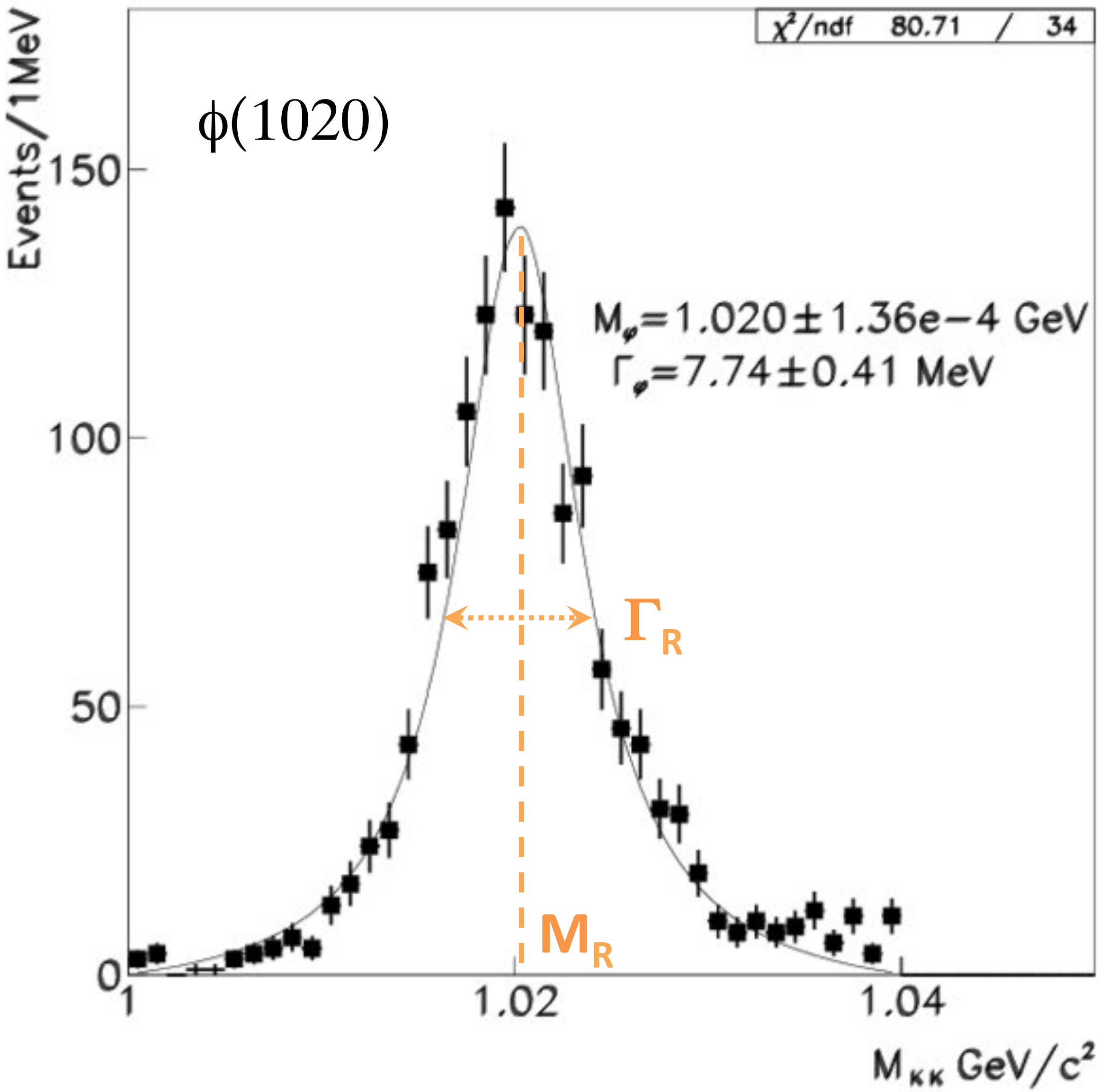
Fitting data with Breit-Wigner (BW) formula

(sophistications : relativistic form, energy dependent width)

$$\frac{1}{E - M_R + i\frac{\Gamma_R}{2}}$$

M_R : (BW) resonance mass, central position of bump structure

Γ_R : (BW) resonance width, FWHM (full width at half maximum) of bump structure



χ^2/ndf 80.71 / 34
φ(1020)
M_φ=1.020±1.36e−4 GeV
Γ_φ=7.74±0.41 MeV
Γ_R
M_R
Events/1MeV
150
100
50
0
1
1.02
1.04
M_{KK} GeV/c^2

BW model : simple and practical method to extract resonance properties,

but approximate method

Common criticism on BW model for missing physical principles, kinematical effects

- Unitarity is not satisfied
- Threshold effects not considered

← Discussed in this lecture

- Analyticity
- Crossing symmetry
- Gauge invariance (electromagnetic process)

→ Important to understand BW's applicability limit and how to overcome

1. Understand more proper method that accounts for some of the above points
   (In general case, an exactly correct method does not exist)
2. Understand how the BW is related to the proper method,
   where BW can be used, and how BW can be improved

1. What is resonance (state) ?

How "state" is formally defined in quantum mechanics ?

→ Solution of Schrödinger (eigenvalue) equation

① bound state

boundary condition

$$H\Psi_a = E_a \Psi_a \qquad \Psi_a(r) \to 0 \quad \text{for} \quad r \to +\infty$$

Ψ_a : eigen vector (wave function)

E_a : real eigenvalue (binding energy, discrete spectrum)

② Scattering state

boundary condition

$$H\Psi_a = E_a \Psi_a \qquad \Psi_a(r) \to e^{i\vec{p}\cdot\vec{r}} + f\frac{e^{ipr}}{r} \quad \text{for} \quad r \to +\infty$$

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② Scattering state

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Ψ_a : eigen vector (wave function)

E_a : real eigenvalue (scattering energy, continuum spectrum)

③ Resonance state

Outgoing boundary condition

$$H\Psi_a = E_a \Psi_a \qquad \Psi_a(r) \rightarrow \frac{e^{ipr}}{r} \quad \text{for} \quad r \rightarrow +\infty$$

Ψ_a : eigen vector (wave function)

$E_a = M_a - i\,\Gamma_a/2$: complex eigenvalue (discrete spectrum)

2. How to extract resonance and its properties from data ?

Resonance properties of interest :

mass, width, decay strength to continuum channels (branching ratios)

Definition of resonance state: $$H\Psi_a = E_a\Psi_a$$

(outgoing boundary condition)

What to do:

① Determine Hamiltonian H that describes data well

Solve the above eigenvalue equation with real energy,

under bound or scattering state boundary conditions

② Search for a resonance

Solve the above eigenvalue equation with complex energy

under outgoing boundary condition

What to do in (more) practice

① Determine Hamiltonian H that describes data well

→ Discussion on how to solve the eigenvalue (scattering) equation

Properties of T-matrix

Relation with S-matrix, unitarity

Alternative approach: K-matrix

Searching for a pole

T(E) on two (physical or unphysical) Riemann sheets

T(E) has two different values for a given E (two valuedness)

→ T(E) is a two-valued function of E

but $T(p_0)$ is a single-valued function of p_0

This occurs because $T(p_0) \neq T(-p_0)$ and $E = (p_0)^2/2\mu = (-p_0)^2/2\mu$

How to differentiate two values of T(E) ?

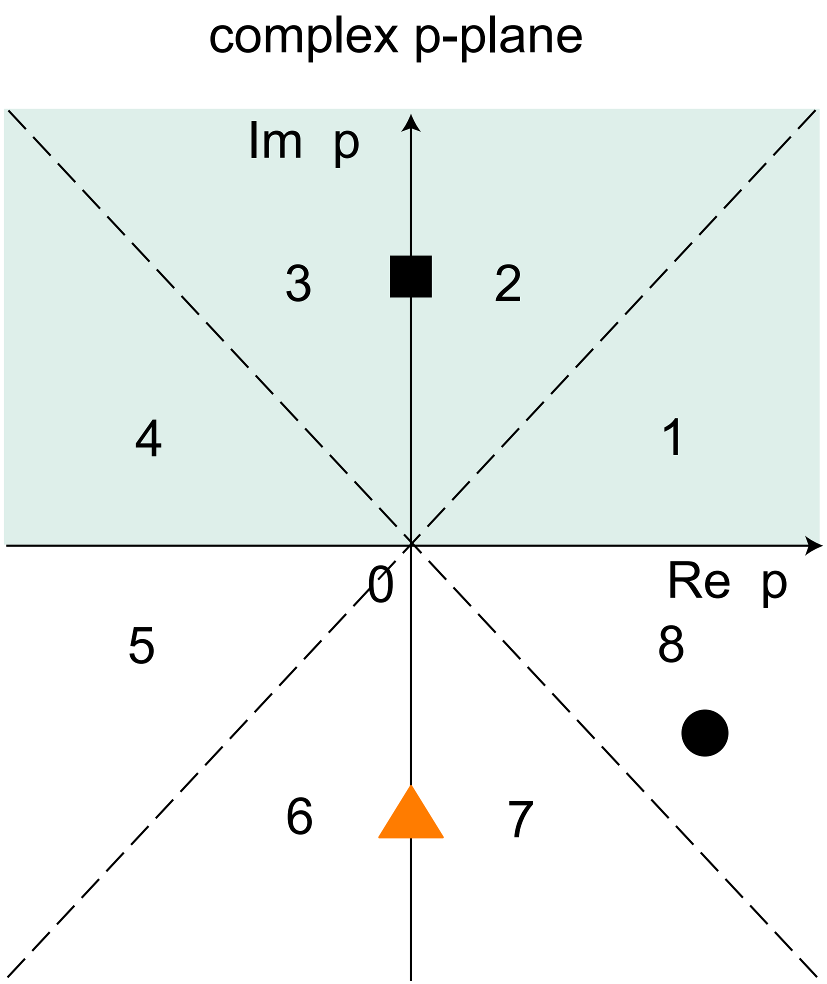
Introduce two Riemann sheets: physical and unphysical sheet

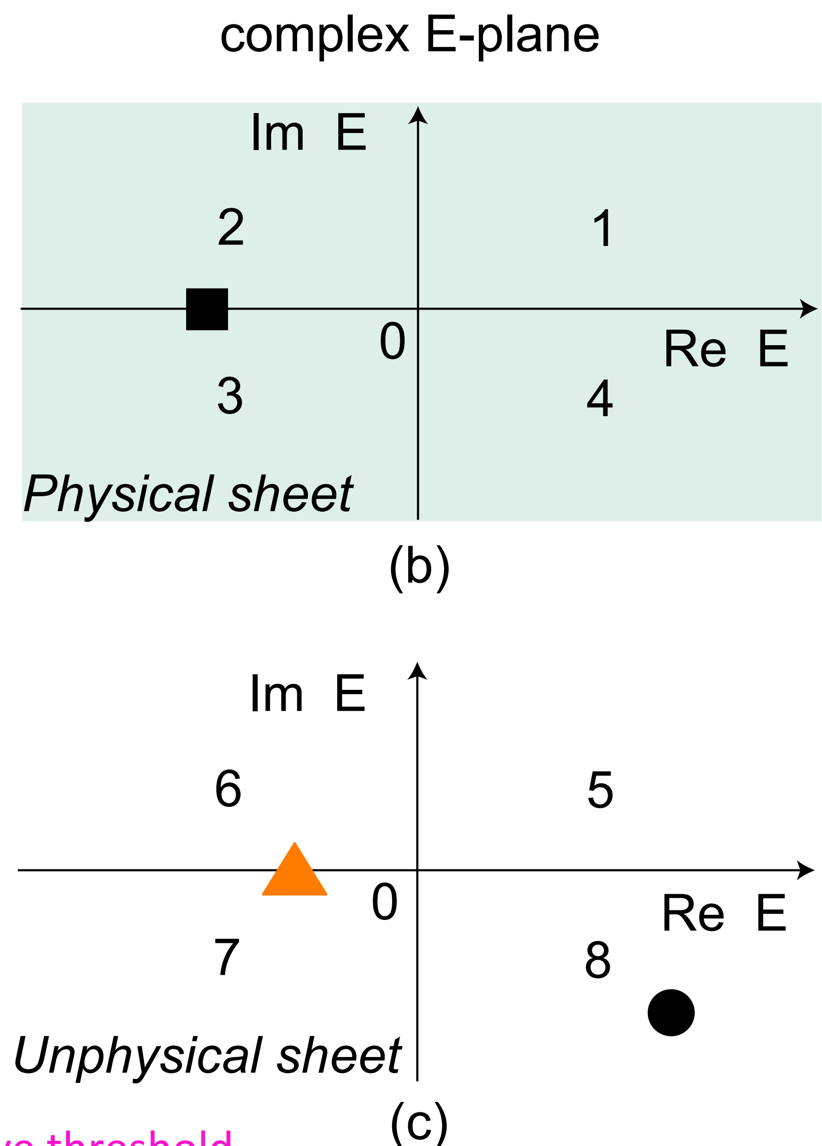
If $\mathrm{Im}[p] \geq 0$ (< 0), $E = p^2/2\mu$ is on physical (unphysical) sheet

If a pole is found at p_R with $\mathrm{Im}[p_R] \geq 0$ (< 0),

the pole is located on physical (unphysical) sheet

T(E) on physical or unphysical sheets





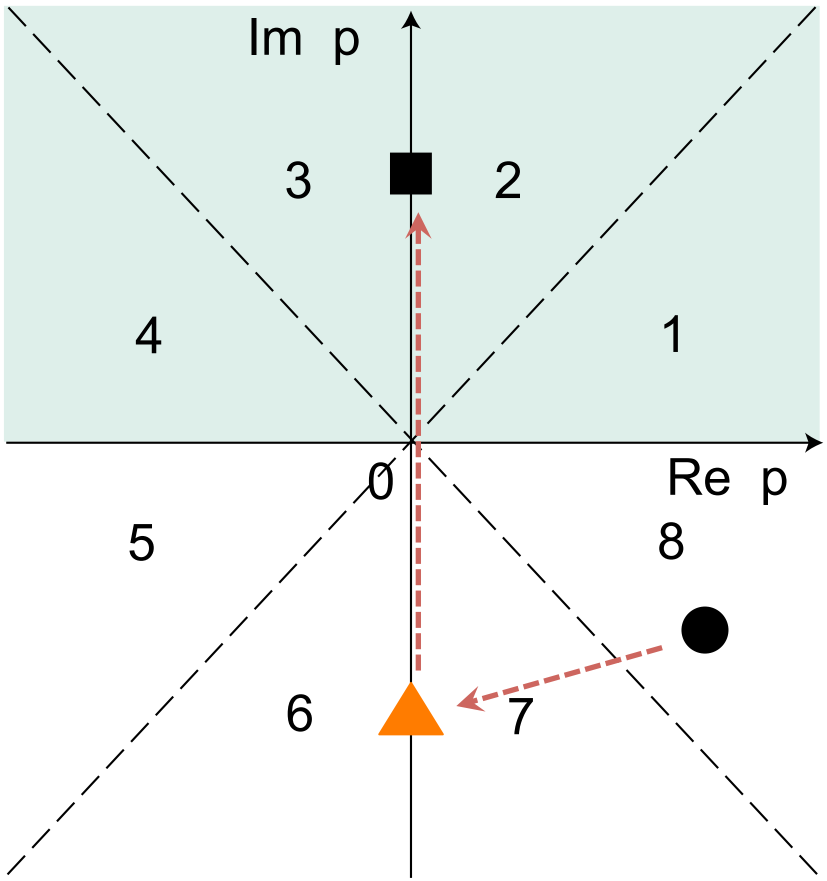
■ : bound state pole is on physical sheet

● : resonance pole is on unphysical sheet above threshold

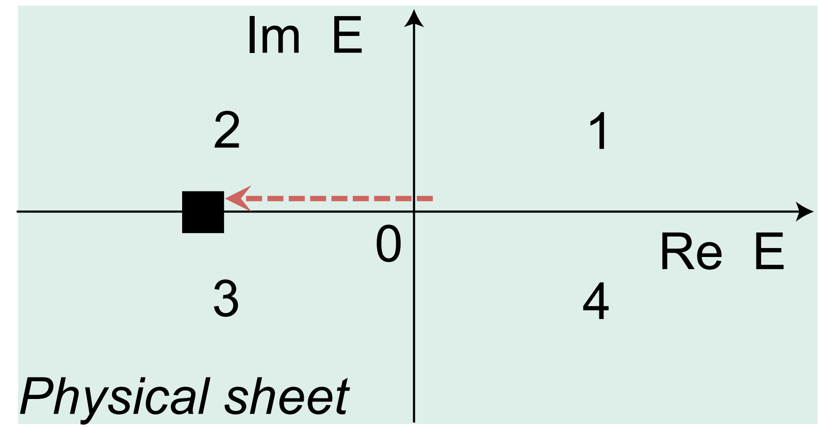
▲ : virtual state (shadow) pole is on unphysical sheet below threshold

Typical trajectory of pole for more attractive interaction

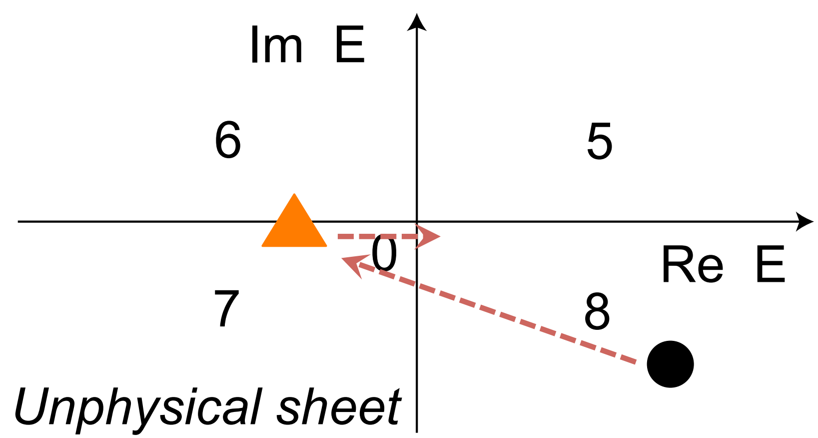
complex p-plane



complex E-plane



(b)



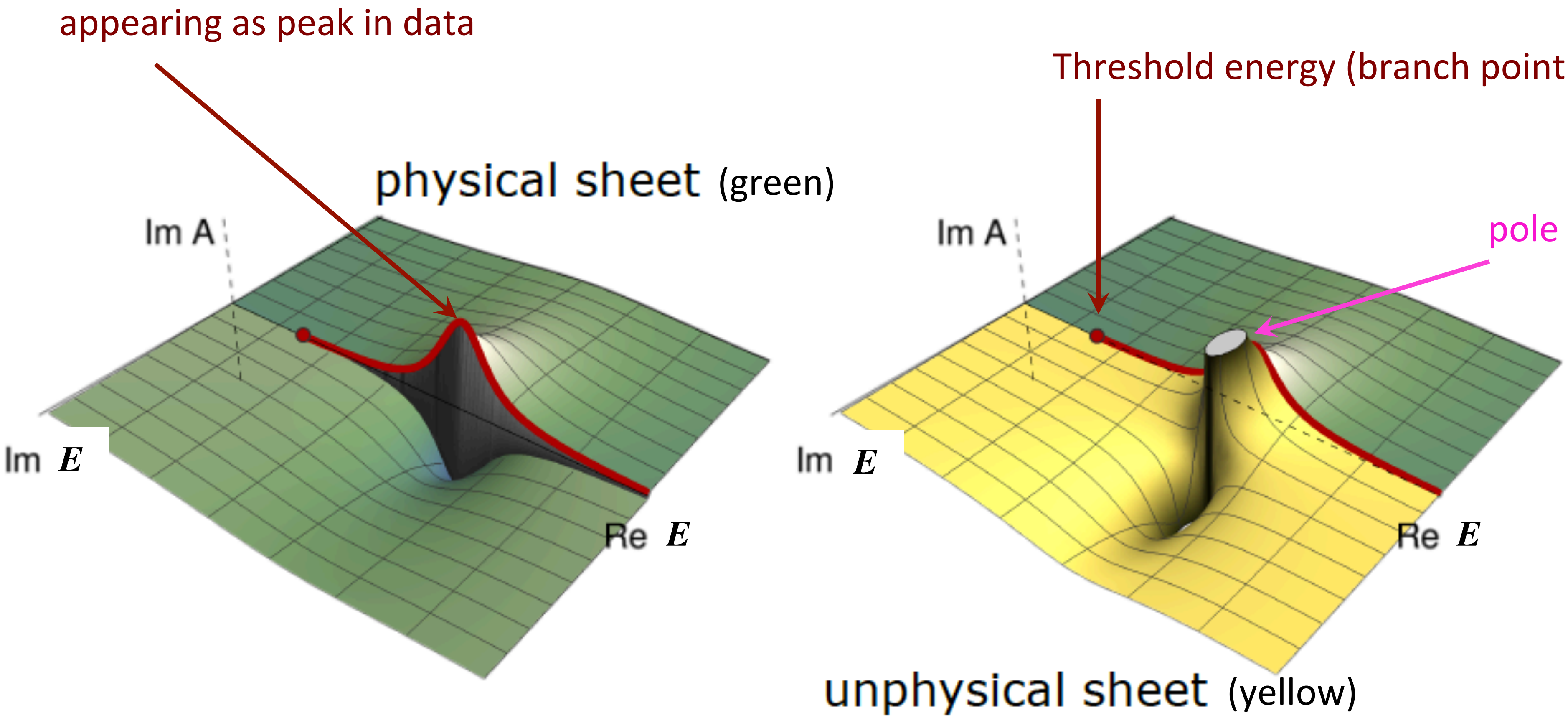
(c)

: bound state pole is on physical sheet

: resonance pole is on unphysical sheet above threshold

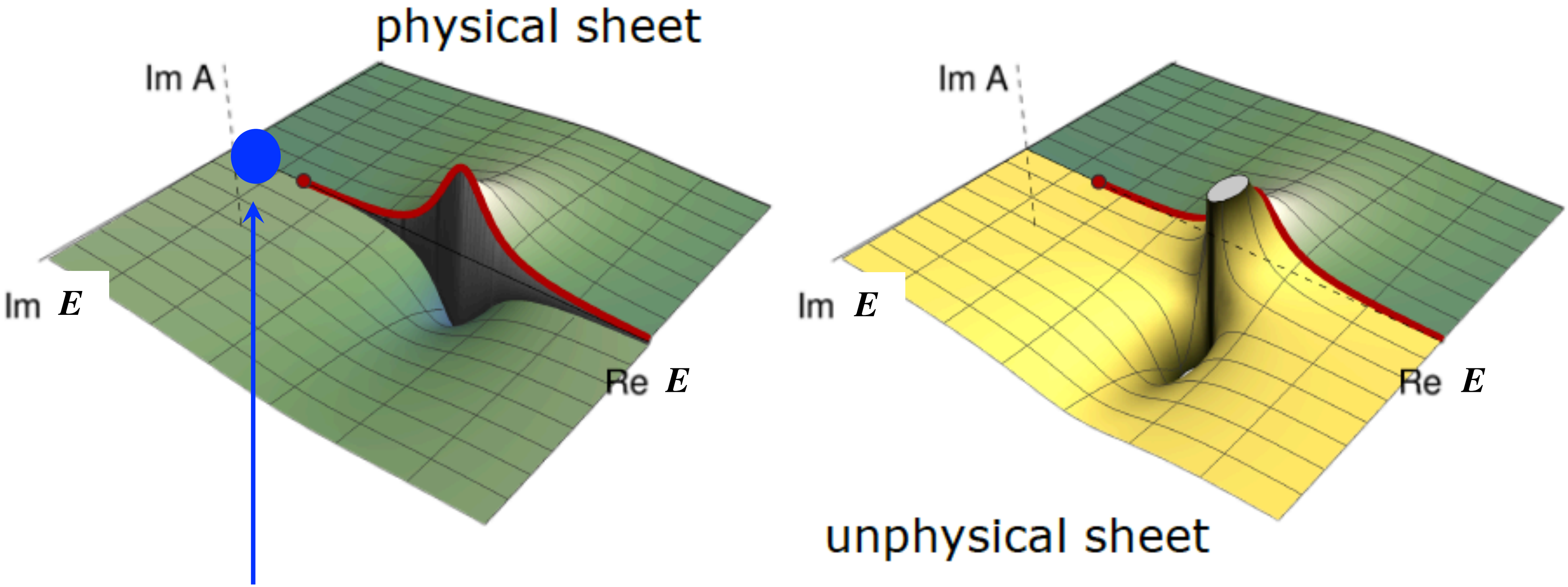
: virtual state (shadow) pole is on unphysical sheet below threshold

T(E) on two Riemann sheets and resonance pole on unphysical sheet



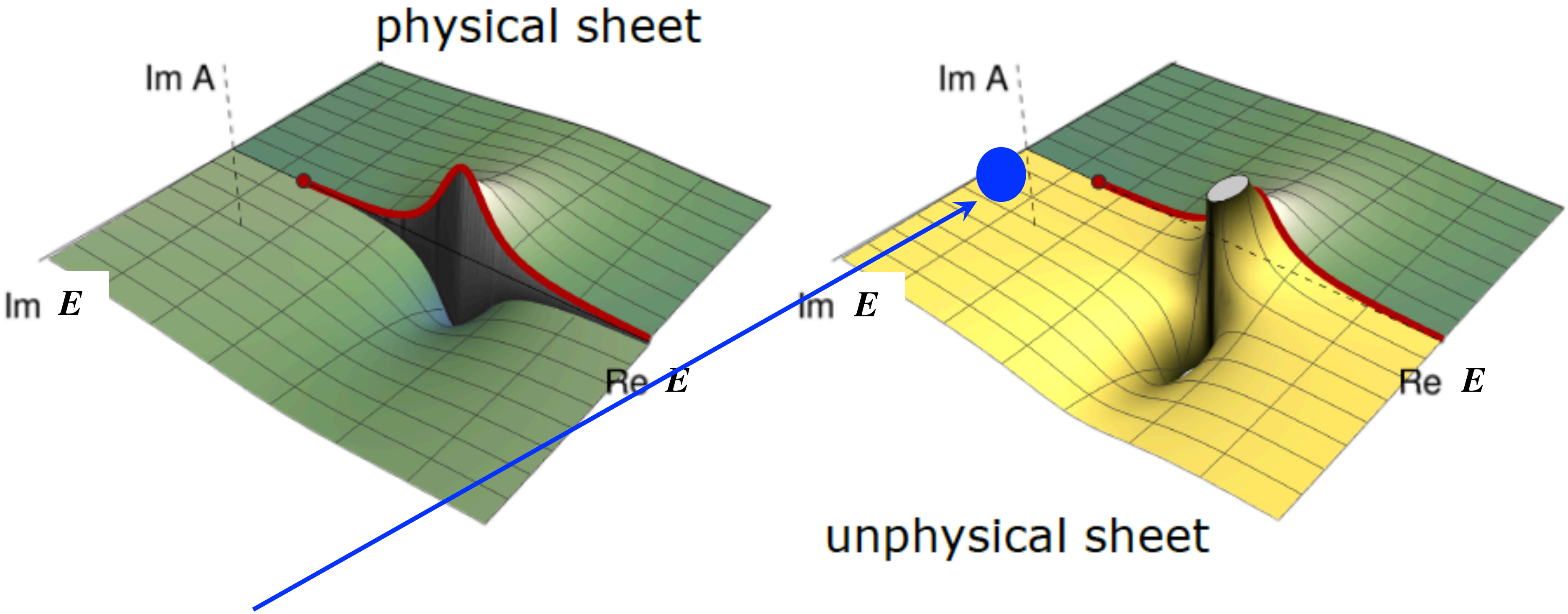
Re E axis above branch point is branch cut where physical and unphysical sheets are smoothly connected

Quiz #1



Q. If a pole exists here, is this (bound, virtual, resonance) pole ?

Quiz #2



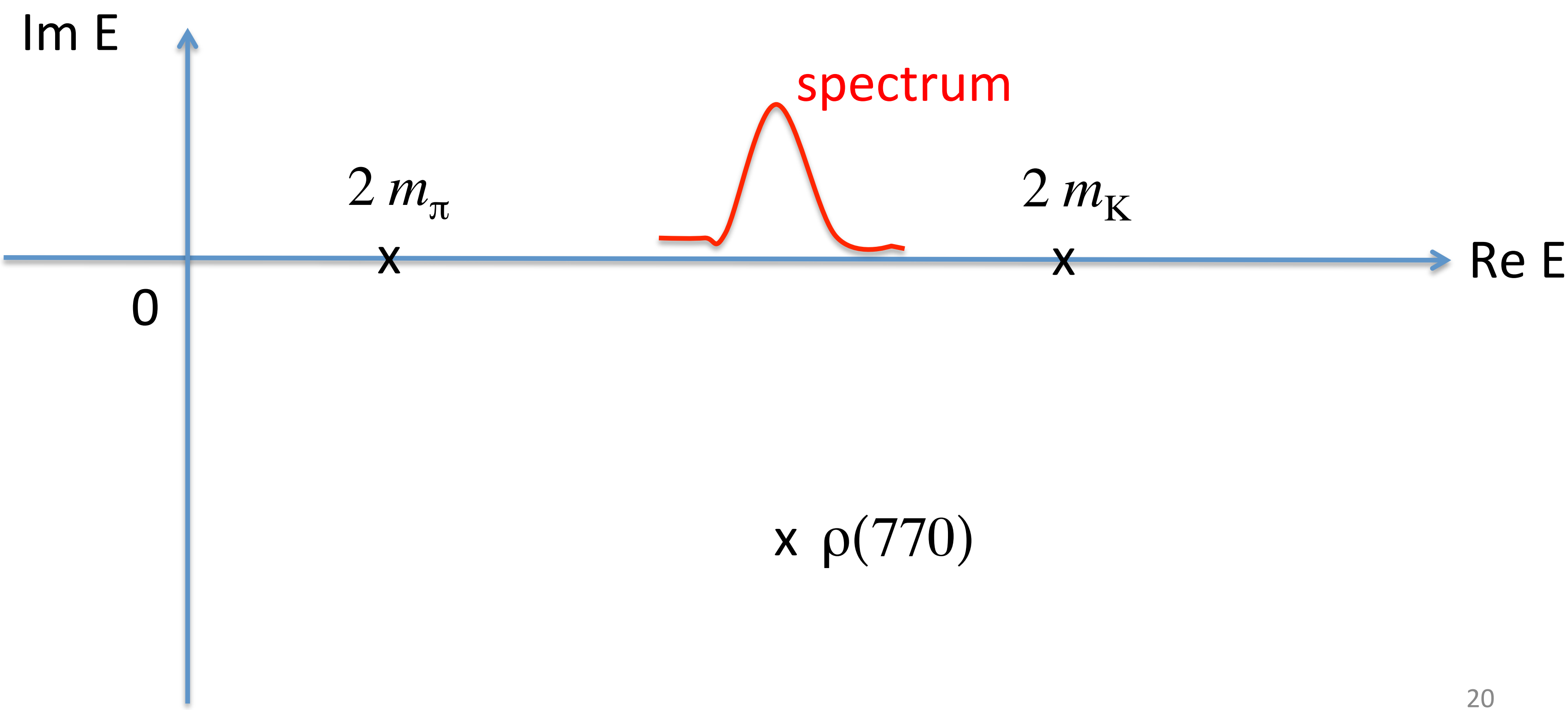
Q. If a pole exists here, is this (bound, virtual, resonance) pole ?

Riemann sheet for coupled-channel scattering

Quiz #3

Which sheet is $\rho(770)$ located on ?   Sheet I, II, III, IV ?

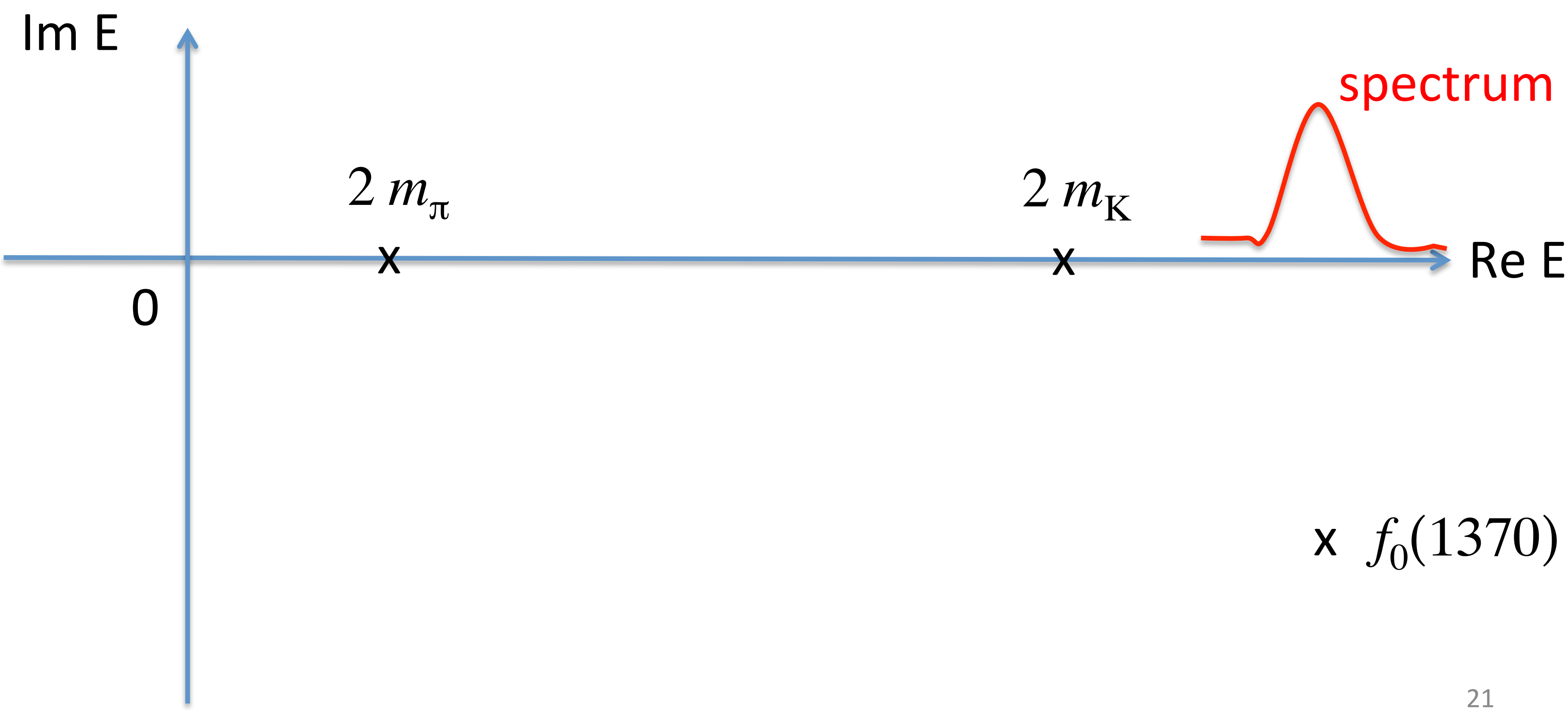
$\rho(770)$: $\pi\pi$ p-wave resonance at ~(770 – 75i) MeV



Quiz #4

Which sheet is $f_0(1370)$ located on ?   Sheet I, II, III, IV ?

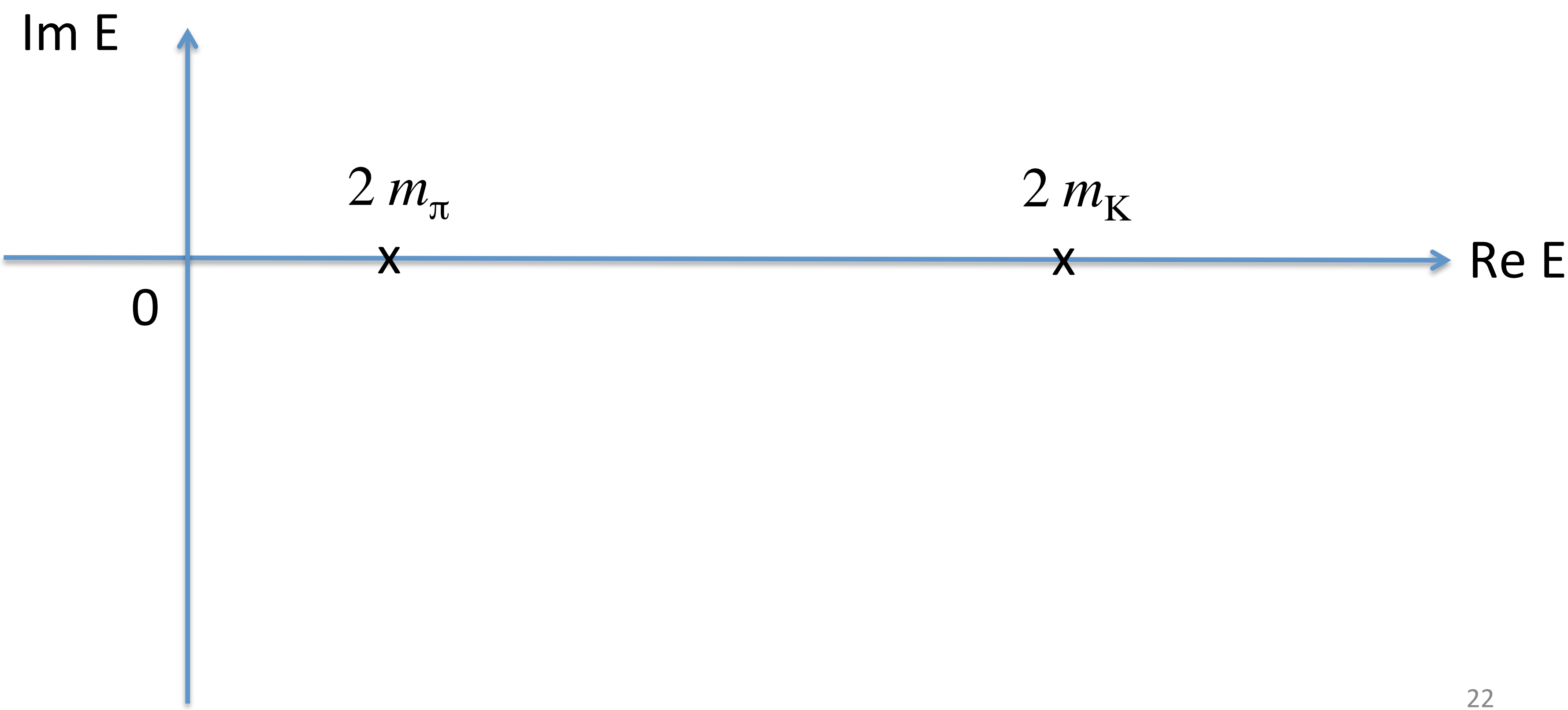
$f_0(1370)$: ππ s-wave resonance at ~(1370 – 150i) MeV



Quiz #5

$f_0(980)$ may be $K\bar{K}$ bound state.

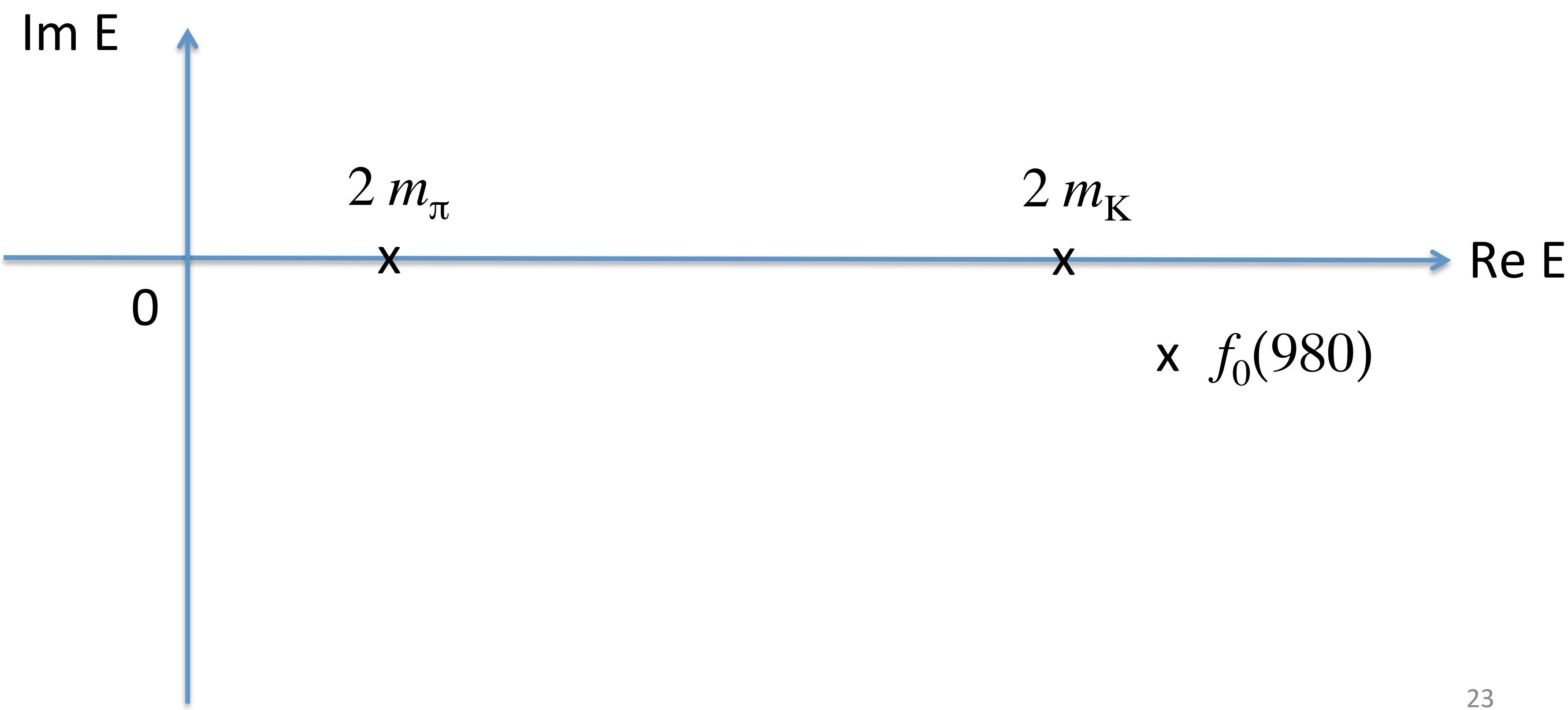
If so, which sheet is $f_0(980)$ located on ? Sheet I, II, III, IV ?



Quiz #6

One analysis found $f_0(980)$ pole at (1030 - 35i) MeV of sheet II.

(Roughly) Indicate energy region where $\pi\pi$ scattering cross section is most influenced by this pole.



Conditions for BW model to work well

1. Isolated resonance (no overlapping with other resonances)
2. Width is narrow
3. Far from relevant thresholds

Use of Flatté model in LHCb analysis on X(3872)

LHCb, *Phys.Rev.D* 102, 092005 (2020)

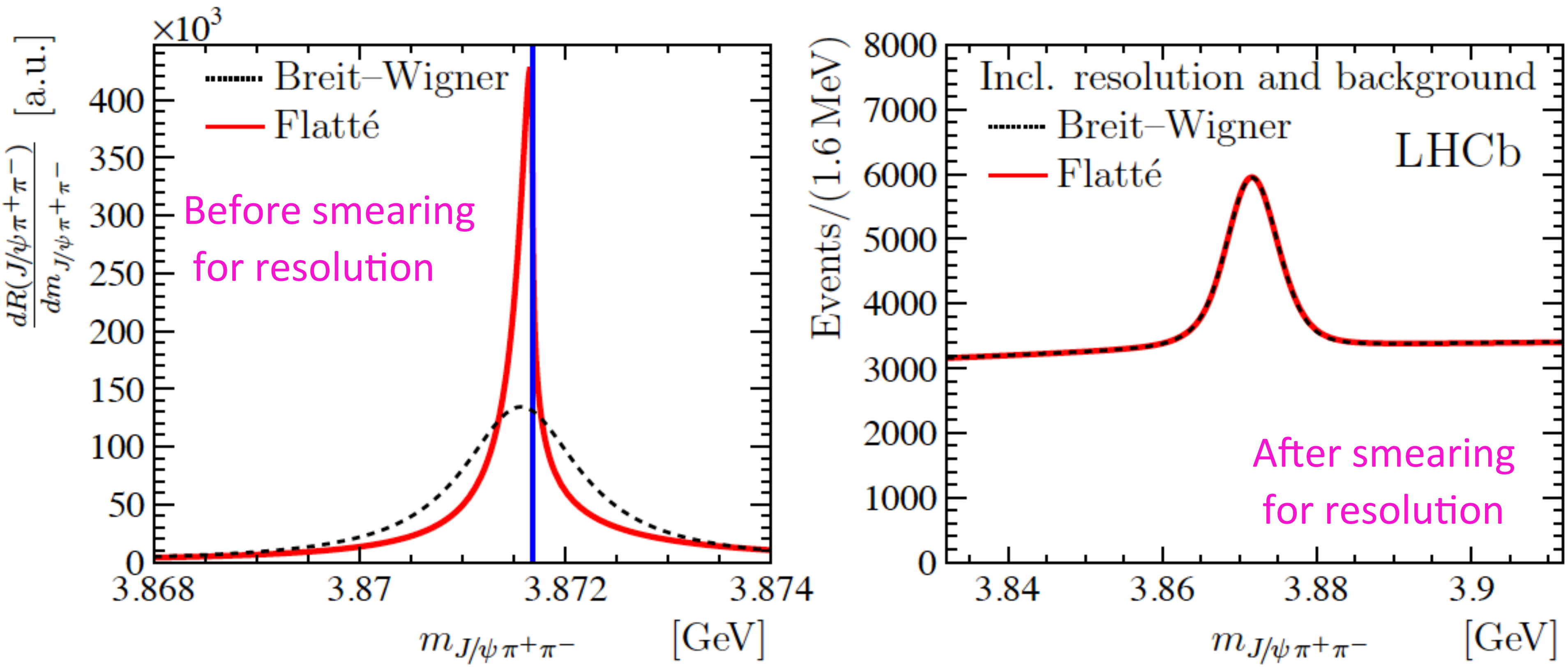
X(3872) mass is very close to $D^{*0}\bar{D}^0$ threshold, candidate of $D^{*0}\bar{D}^0$ molecule

$$m_{X(3872)} \approx m_{D^{*0}} + m_{D^0}$$

→ Bad condition for BW model

Comparison of Flatté and BW models

LHCb, *Phys.Rev.D* 102, 092005 (2020)



- Flatté model has asymmetric and shaper lineshape
- After smearing, both Flatté and BW models give essentially the same lineshape
- FWHM (full width at half maximum) is rather different

  Flatté : 0.2 MeV , BW : 1.4 MeV

  → extracted X(3872) width significantly depends on model (Flatté is more reliable)

3. Fake of resonance

Experimentally, peak structure is searched for in energy spectrum

because it would be often a signal from a resonance (peak hunt).

However, peak is not always from a resonance,

and sometimes mistakenly interpreted as a resonance.

Such fake resonance structures appear most often near thresholds

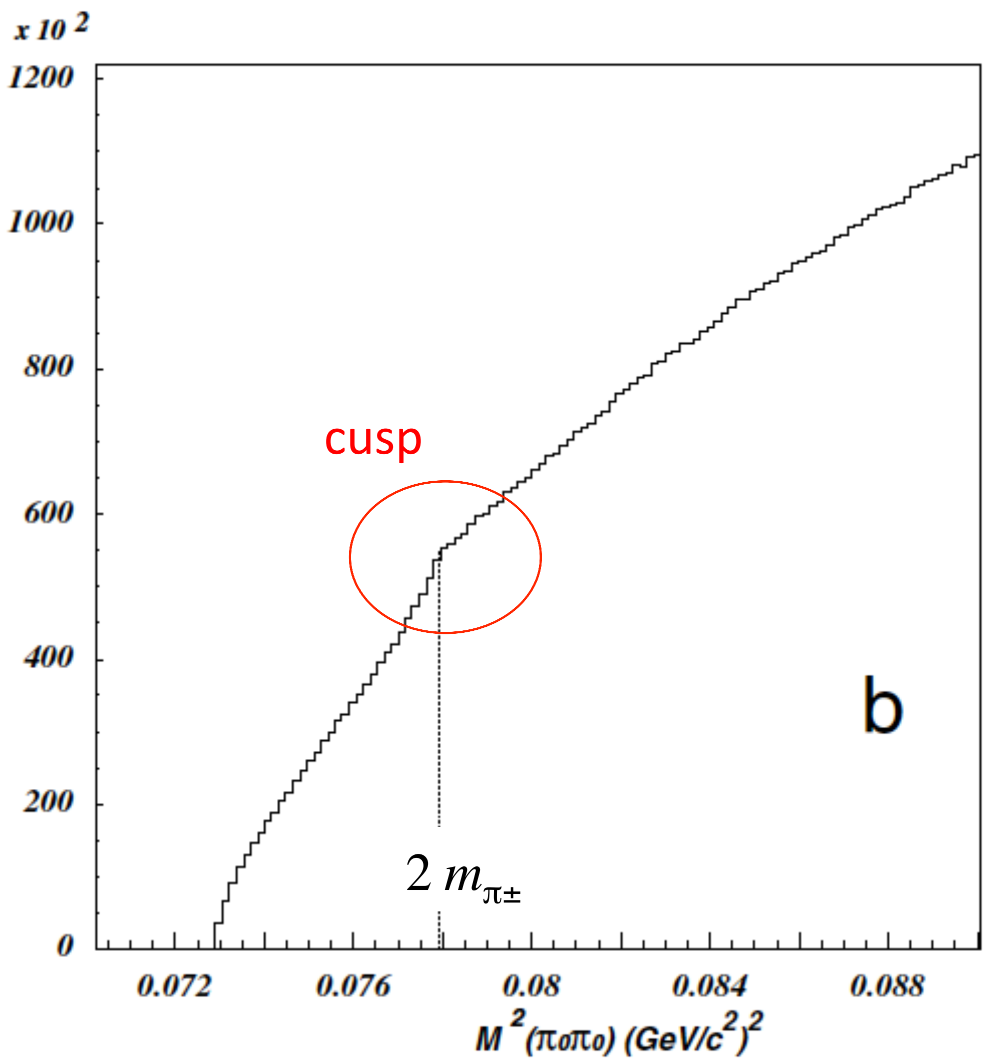
Two common mechanisms for fake resonance structures

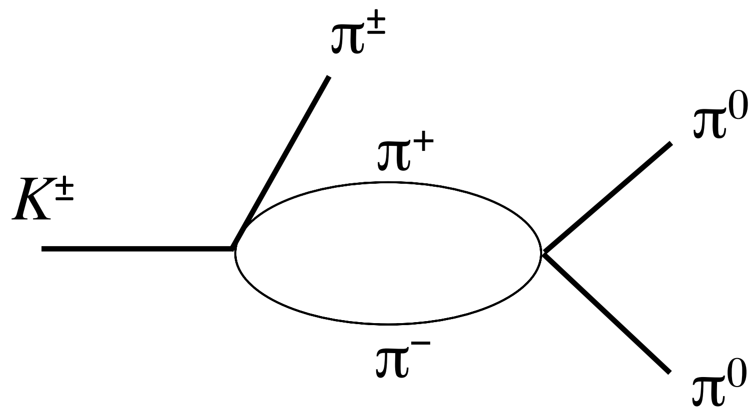
1. Threshold cusp
2. Triangle singularity

3.1 Threshold cusp

Example of threshold cusp : 1. pion-pion scattering from $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

NA48, Nuclear Physics B (Proc. Suppl.) 164, 85 (2007)



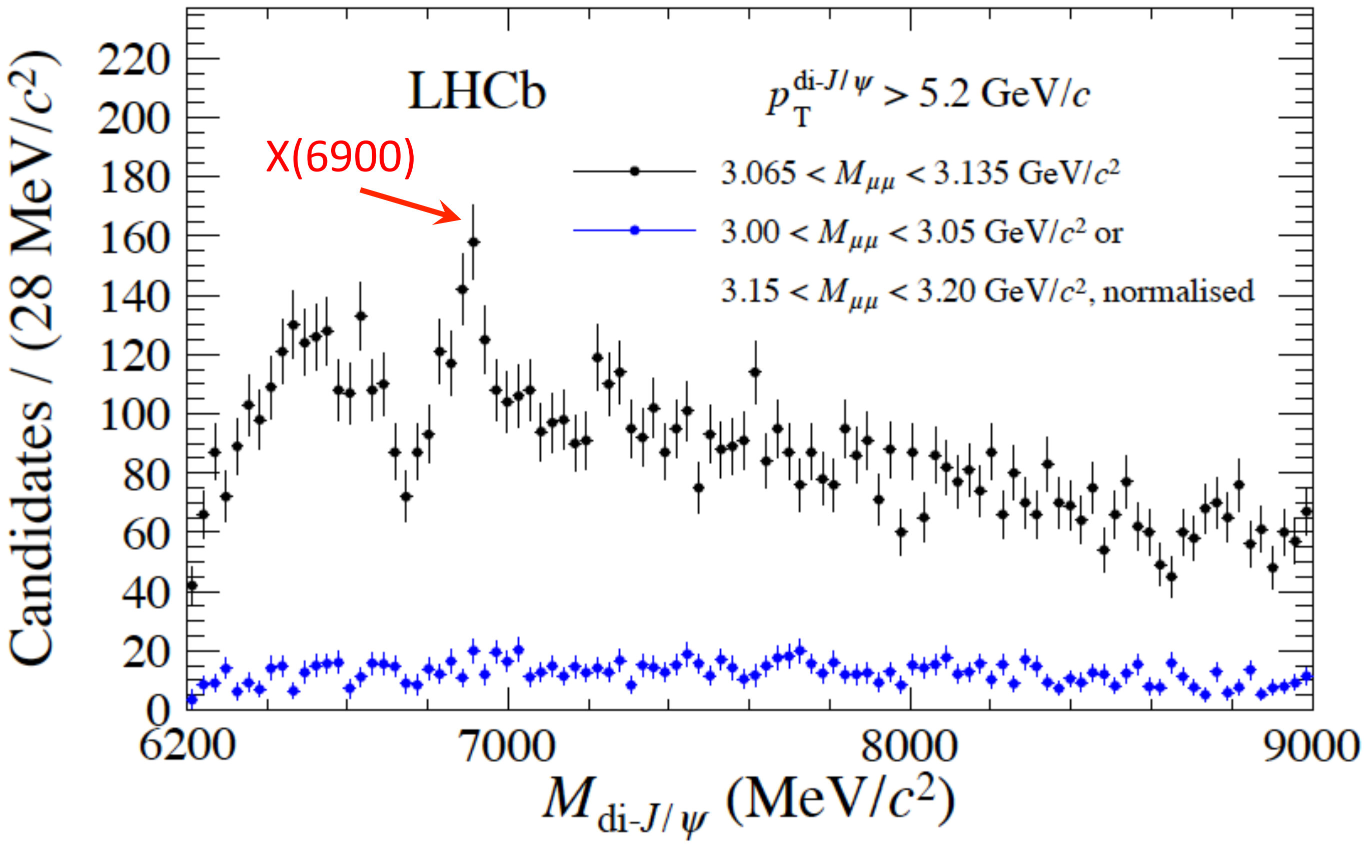


By analyzing the cusp, pion-pion scattering length was precisely determined

Well-established case of threshold cusp effect

Example of threshold cusp : 2. J/ψ pair mass spectrum from *pp* collision

LHCb, Science Bulletin 65, 1983 (2020)

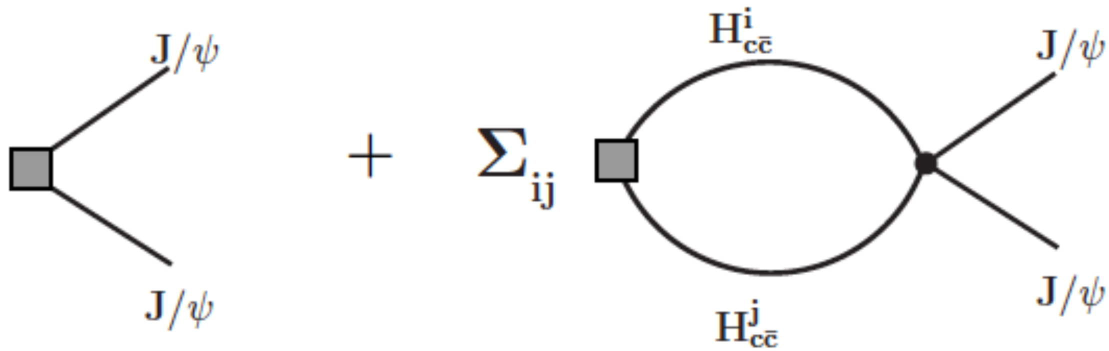


X(6900) was claimed as a new resonance (tetra charm quark)

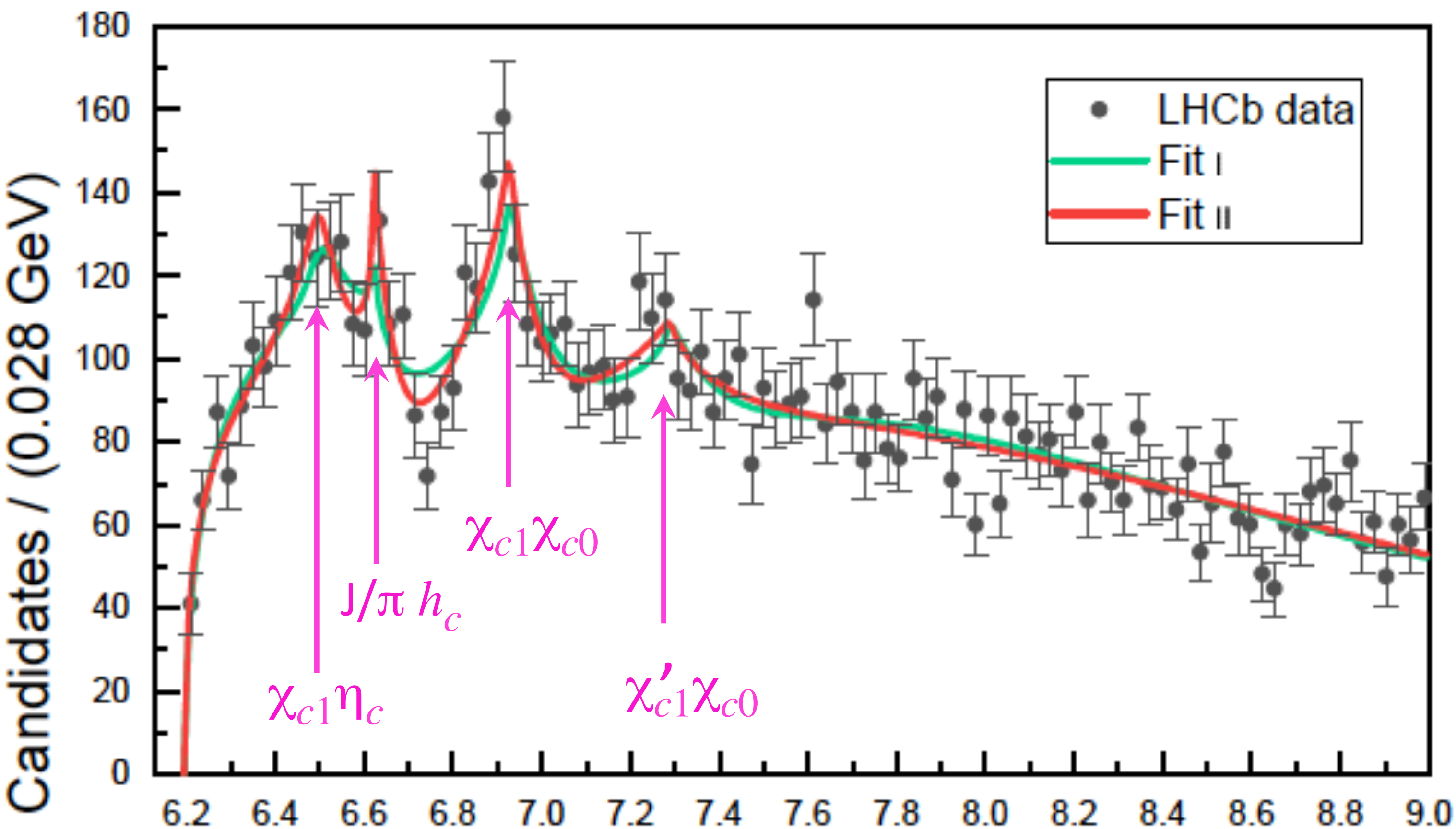
→ Many theoretical papers support this interpretation

Threshold cusp may explain the LHCb data

Wang et al. arXiv:2008.07430v2



Still under active debate



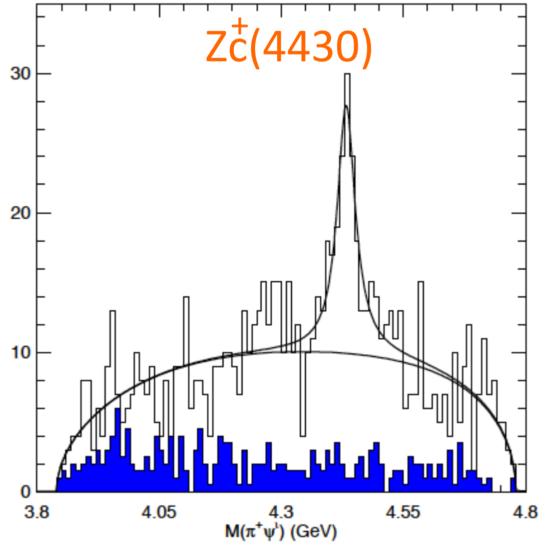
3.2 Triangle singularity (TS)

Most useful paper to understand TS:
PRD 94, 074039 (2016)

Example of TS : Zc(4430)

There is no established case of TS in experimental data

Belle (2008)



$$\overline{B}^0 \to \psi(2S)\pi^+K^-$$

If charged quarkonium-like states $Z_c^+ : c\bar{c}u\bar{d}$

Minimally 4-quark states and not $q\bar{q}$

→ clear signature of exotics

$Z_c(4430)$ has been outstanding exotic candidate

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LHCb confirms existence of exotic hadrons

The LHCb collaboration today published an unambiguous observation of an exotic particle that cannot be classified within the traditional quark model

9 APRIL, 2014 | By Cian O'Luanaigh

The Large Hadron Collider beauty (LHCb) collaboration today announced results that confirm the existence of exotic hadrons – a type of matter that cannot be classified within the traditional quark model.

Hadrons are subatomic particles that can take part in the strong interaction – the force that binds protons inside the nuclei of atoms. Physicists have theorized since the 1960s, and ample experimental evidence since has confirmed, that hadrons are made up of quarks and antiquarks that determine their properties. A subset of hadrons, called mesons, is formed from quark-antiquark pairs, while the rest – baryons – are made up of three quarks.

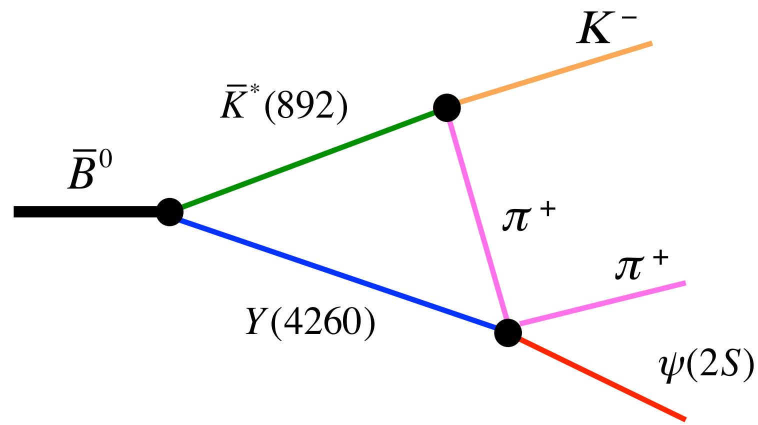
But since it was first proposed physicists have found several particles that do not fit into this model of hadron structure. Now the LHCb collaboration has published an unambiguous observation of an exotic particle – the Z(4430) – that does not fit the quark model.

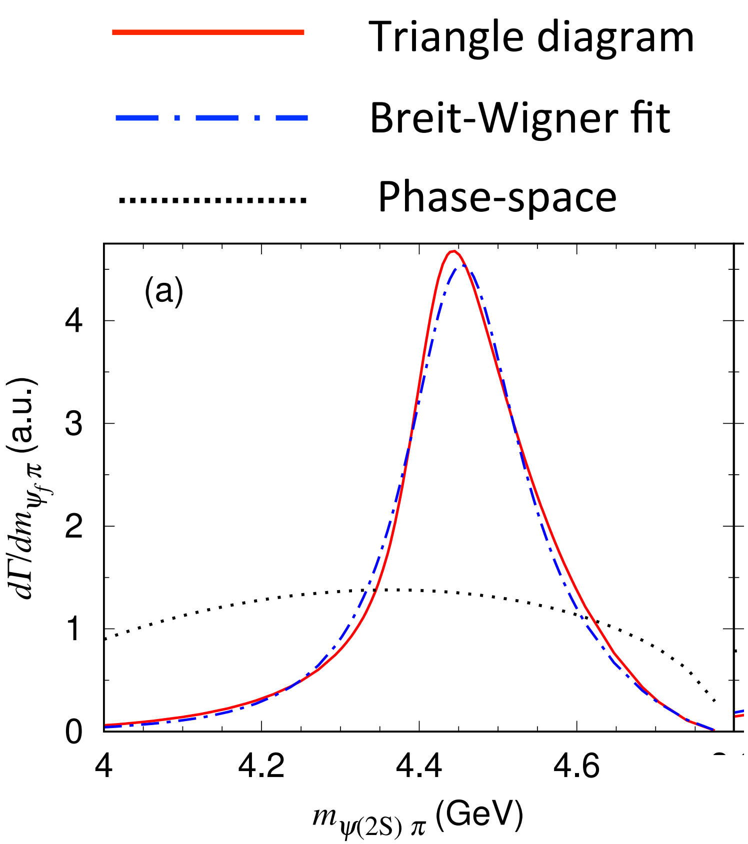
Related Articles

https://home.cern/news/news/experiments/lhcb-confirms-existence-exotic-hadrons

Triangle singularity for Zc(4430)

SXN and Tsushima,
PRD 100, 051502(R) (2019)

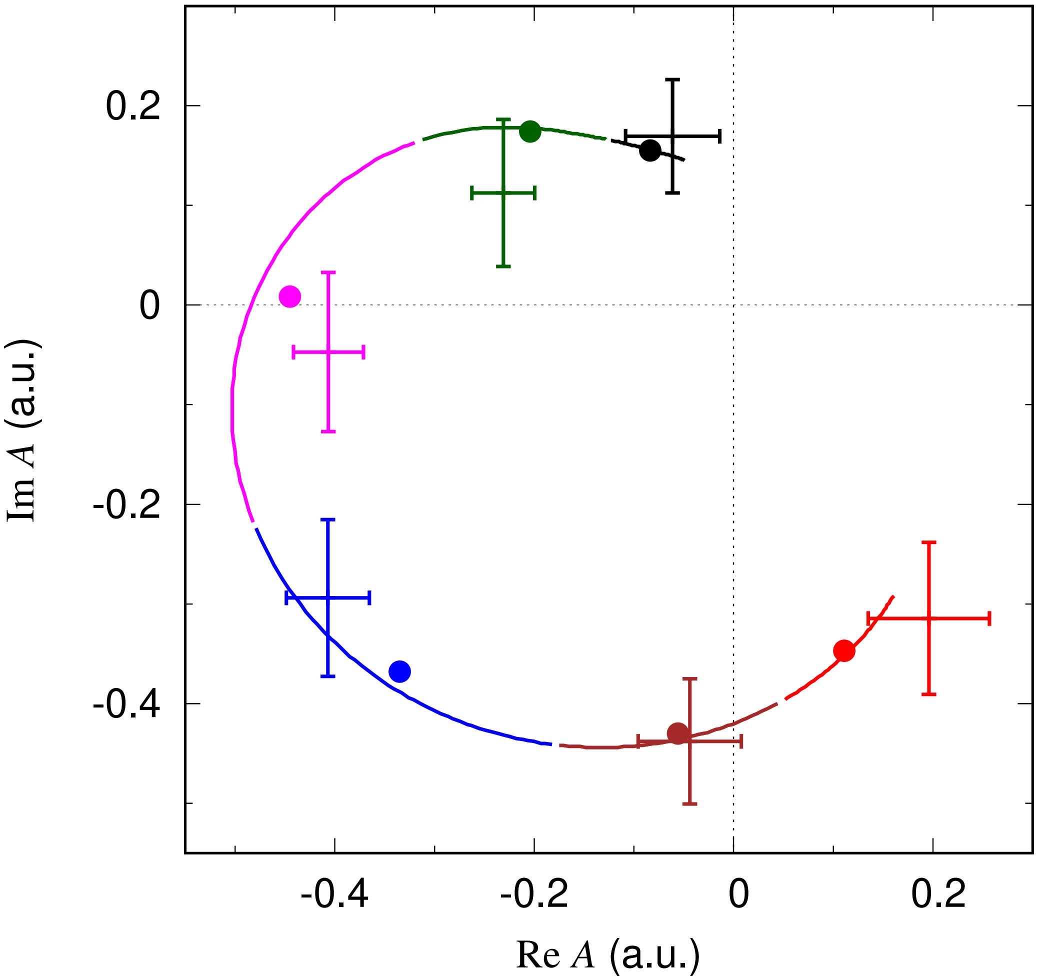




	TS model	Belle (2013)	LHCb (2014)
Mass (MeV)	4463 ± 13	$4485 \pm 22^{+28}_{-11}$	$4475 \pm 7^{+15}_{-25}$
Width (MeV)	195 ± 16	200^{+41+26}_{-46-35}	$172 \pm 13^{+37}_{-34}$

Still absolute strength is question

Zc(4430) Argand plot



Resonance-like counter-clockwise motion is reproduced by triangle diagram, not a resonance

Curved segment and point of same color belong to same $m^2_{\psi(2S)\pi^+}$ bin

Data: LHCb, PRL 112 222002 (2014)

END